

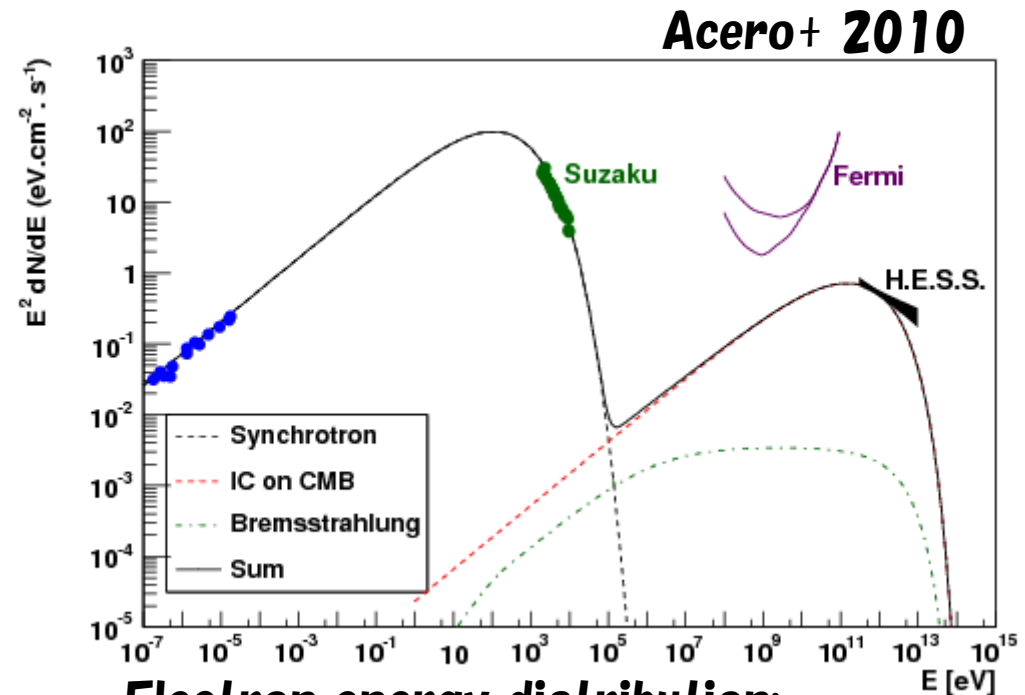
# ***Stochastic Particle Acceleration via Turbulence in Various High-Energy Astrophysical Phenomena***

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**Katsuaki Asano  
(ICRR)**

# Non-thermal Emission

## Supernova Remnant SN1006



**Electron energy distribution:**  
**Single power-law with the index of 2.1,**  
**cut off at 10 TeV and  $B=30 \mu\text{G}$ .**

$$t_{\text{acc}} = \frac{20\eta E}{3eBc\beta_{\text{SNR}}^2} \quad \& \quad t_{\text{cool}} = \frac{6\pi m_e^2 c^3}{\sigma_T B^2 E}$$

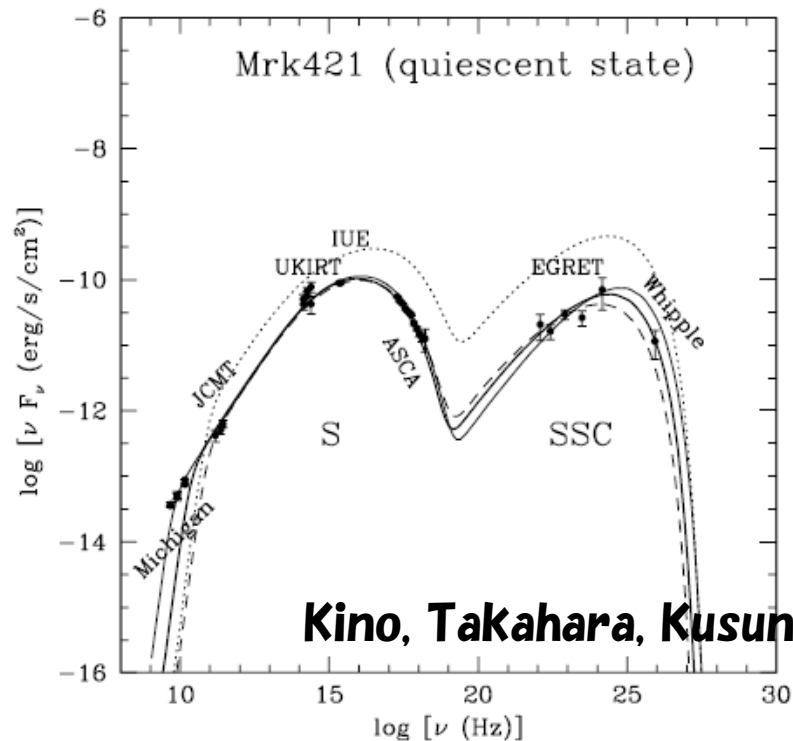
**Bohm Limit** ( $\eta = 1$ ) &  $v_{\text{SNR}} = \frac{1000\text{km}}{\text{s}} \rightarrow E_{\text{max}} = 8\text{TeV}$

**Consistent with the diffusive shock acceleration (Fermi I)+Bohm Limit!**

**All non-thermal phenomena are due to shocks?**

# Troubles in Blazar Emission

**1. Index harder than 2 for the electron injection spectrum.**



**Kino, Takahara, Kusunose 2002**

FIG. 4.—One-zone SSC model spectra for the steady state emission of Mrk 421. The thick solid line shows the best-fit spectrum where the adopted parameters are  $\delta = 12$ ,  $R = 2.8 \times 10^{16}$  cm,  $B = 0.12$  G,  $\gamma_{\max} = 1.5 \times 10^5$ ,  $q_e = 9.6 \times 10^{-6}$  cm $^{-3}$  s $^{-1}$ ,  $s = 1.6$ , and  $u_e/u_B = 5$ . The dotted line shows the spectrum obtained using the analytic estimates for Mrk 421. The thin solid and dashed lines show the spectra of low and high injection models, respectively, to indicate the uncertainty range of the spectral fitting.

**2. Lower maximum energy**

**Mrk 421:**

**$B = 38$  mG**

**If  $\eta = 1$ , even for  $\beta_{\text{sh}} = 0.1$ ,**

**$E_{\max} = 7$  TeV.**

**But actual Max. Energy 50 GeV**

**The Bohm factor should be  $\sim 10^4$**

**Inoue & Takahara 2002**

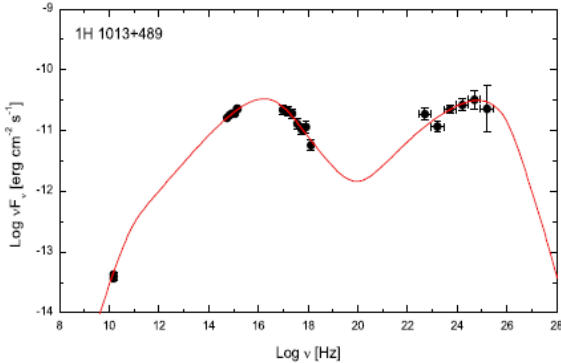
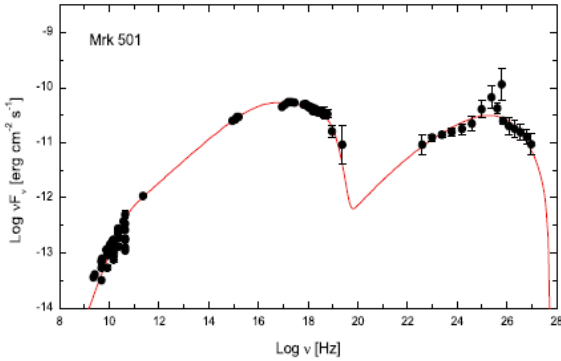
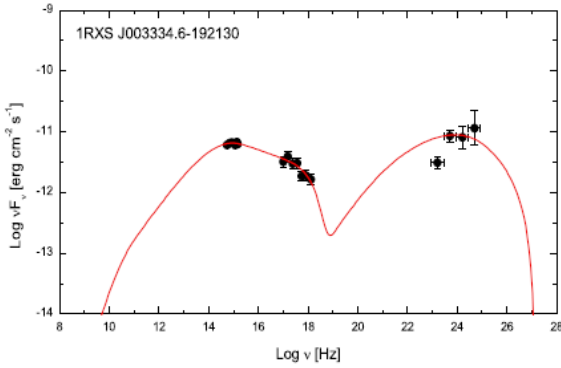
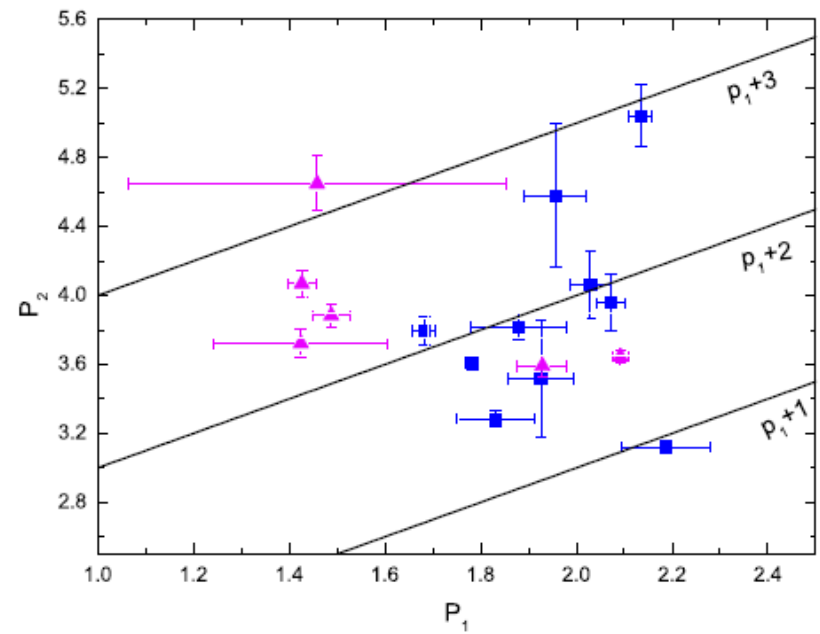
**3. Too sharp break in the electron spectrum.**

**Compared to the cooling break, the difference in the indices seems large.**

# Electron Index in Fermi BL Lacs

Yan+

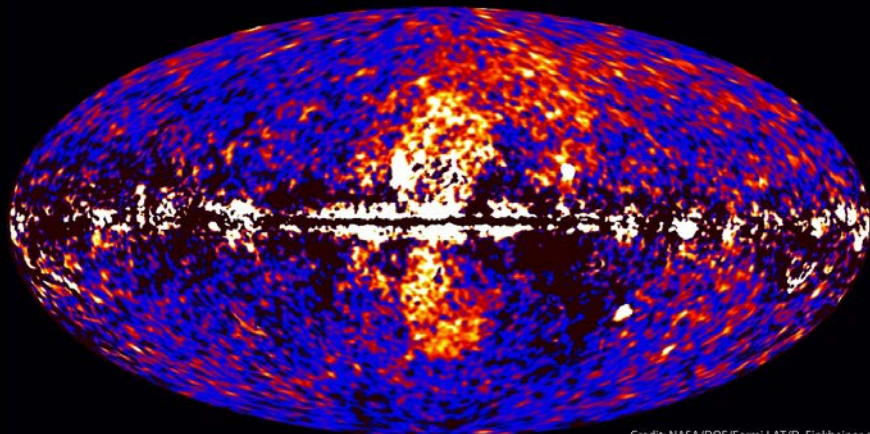
MNRAS 439, 2933–2942 (2014)



Name	$B$ (0.01 G)	$\delta_D$ (10)	$t_{\nu, \min}$ ( $10^5$ s)	$\gamma'_{\max}$ ( $10^7$ )	$\gamma'_b$ ( $10^4$ )	$K'_e$ ( $10^{55}$ )	$p_1$	$p_2$	$\chi^2_{\text{red}}$
0033–1921	$4.06 \pm 1.24$	$2.43 \pm 0.17$	$2.48 \pm 1.21$	$0.07 \pm 0.01$	$1.62 \pm 0.20$	$0.12 \pm 0.01$	$1.83 \pm 0.08$	$3.29 \pm 0.05$	1.14
0414+009	$1.30 \pm 0.58$	$2.96 \pm 1.36$	$3.54 \pm 4.31$	$1.49 \pm 2.70$	$12.67 \pm 1.36$	$0.04 \pm 0.02$	$1.88 \pm 0.10$	$3.82 \pm 0.07$	3.96
0447–439	$5.47 \pm 1.38$	$3.63 \pm 0.08$	$0.43 \pm 0.11$	$0.052 \pm 0.002$	$3.18 \pm 0.29$	$0.05 \pm 0.02$	$2.07 \pm 0.03$	$3.96 \pm 0.17$	0.70
1013+489	$5.72 \pm 0.75$	$2.75 \pm 0.47$	$0.55 \pm 0.22$	$0.08 \pm 0.04$	$6.82 \pm 0.74$	$0.03 \pm 0.01$	$2.03 \pm 0.04$	$4.06 \pm 0.19$	2.11
2155–304	$4.89 \pm 0.66$	$1.97 \pm 0.06$	$3.47 \pm 0.52$	$0.087 \pm 0.004$	$3.57 \pm 0.20$	$0.011 \pm 0.002$	$1.68 \pm 0.02$	$3.79 \pm 0.08$	2.48
Mrk 421	$4.23 \pm 0.41$	$2.71 \pm 0.27$	$0.42 \pm 0.10$	$3.73 \pm 0.81$	$18.43 \pm 0.79$	$0.012 \pm 0.002$	$2.13 \pm 0.02$	$5.04 \pm 0.18$	1.39
Mrk 501	$2.77 \pm 0.63$	$2.99 \pm 0.70$	$0.16 \pm 0.11$	$0.16 \pm 0.03$	$15.81 \pm 3.10$	$0.007 \pm 0.006$	$2.19 \pm 0.09$	$3.12 \pm 0.04$	1.29
RBS 0413	$5.48 \pm 1.57$	$2.60 \pm 0.55$	$0.23 \pm 0.11$	$1.29 \pm 0.42$	$9.97 \pm 1.26$	$0.0014 \pm 0.0006$	$1.93 \pm 0.07$	$3.52 \pm 0.34$	1.91
1215+303	$3.49 \pm 0.17$	$3.58 \pm 0.10$	$0.22 \pm 0.02$	$0.27 \pm 0.01$	$1.13 \pm 0.04$	$0.0031 \pm 0.0001$	$1.78 \pm 0.01$	$3.61 \pm 0.04$	1.99
2247+381	$5.45 \pm 1.64$	$3.62 \pm 0.05$	$0.14 \pm 0.05$	$0.10 \pm 0.06$	$8.87 \pm 1.96$	$0.0004 \pm 0.0002$	$1.96 \pm 0.06$	$4.58 \pm 0.42$	0.54
0048–09	$6.50 \pm 5.84$	$2.50 \pm 0.28$	$2.19 \pm 1.74$	$0.10 \pm 0.02$	$0.52 \pm 0.04$	$0.015 \pm 0.002$	$1.42 \pm 0.18$	$3.72 \pm 0.08$	2.90
0716+714	$5.90 \pm 1.23$	$2.71 \pm 0.47$	$3.51 \pm 1.21$	$0.04 \pm 0.01$	$0.92 \pm 0.10$	$0.010 \pm 0.002$	$1.49 \pm 0.04$	$3.88 \pm 0.07$	1.98
0851+202	$4.05 \pm 2.41$	$2.40 \pm 1.10$	$2.43 \pm 3.34$	$0.14 \pm 0.45$	$0.26 \pm 0.10$	$0.13 \pm 0.12$	$1.46 \pm 0.40$	$4.65 \pm 0.16$	1.49
1058+5628	$2.20 \pm 1.14$	$2.40 \pm 0.73$	$1.29 \pm 0.72$	$0.06 \pm 0.03$	$2.61 \pm 0.30$	$0.06 \pm 0.03$	$1.93 \pm 0.05$	$3.59 \pm 0.07$	1.36
1246+586	$8.82 \pm 1.89$	$2.34 \pm 0.34$	$3.06 \pm 0.96$	$0.40 \pm 0.02$	$0.89 \pm 0.08$	$0.006 \pm 0.006$	$1.43 \pm 0.03$	$4.08 \pm 0.08$	1.52
W Comae	$4.91 \pm 0.12$	$2.70 \pm 0.13$	$0.32 \pm 0.04$	$0.06 \pm 0.01$	$1.94 \pm 0.09$	$0.046 \pm 0.002$	$2.09 \pm 0.02$	$3.65 \pm 0.04$	1.75
0426–380	$1.08 \pm 2.42$	$3.53 \pm 4.01$	$0.93 \pm 1.31$	$0.47 \pm 0.02$	$1.77 \pm 0.51$	$0.36 \pm 0.78$	$1.78 \pm 0.51$	$3.58 \pm 0.93$	2.41
0537–441	$2.12 \pm 1.55$	$3.62 \pm 1.54$	$1.51 \pm 1.38$	$0.38 \pm 0.40$	$0.54 \pm 0.09$	$0.20 \pm 0.07$	$1.56 \pm 0.13$	$3.96 \pm 0.06$	5.64
1717+177	$1.79 \pm 0.20$	$3.52 \pm 0.18$	$0.036 \pm 0.005$	$0.013 \pm 0.003$	$1.79 \pm 0.17$	$0.020 \pm 0.001$	$2.12 \pm 0.04$	$3.53 \pm 0.19$	3.97
BL Lac	$1.86 \pm 1.89$	$3.23 \pm 1.70$	$0.95 \pm 0.90$	$0.11 \pm 0.09$	$0.29 \pm 0.06$	$0.24 \pm 0.04$	$1.84 \pm 0.18$	$3.87 \pm 0.04$	4.10
OT 081	$9.82 \pm 9.80$	$2.31 \pm 5.16$	$0.12 \pm 0.55$	$2.00 \pm 2.10$	$0.52 \pm 0.58$	$0.007 \pm 0.022$	$1.75 \pm 0.66$	$3.76 \pm 0.59$	1.69
4C 01.28	$10.56 \pm 19.20$	$2.47 \pm 3.42$	$0.66 \pm 6.02$	$0.12 \pm 0.43$	$0.30 \pm 0.18$	$0.06 \pm 0.13$	$1.69 \pm 0.64$	$3.70 \pm 0.32$	0.96

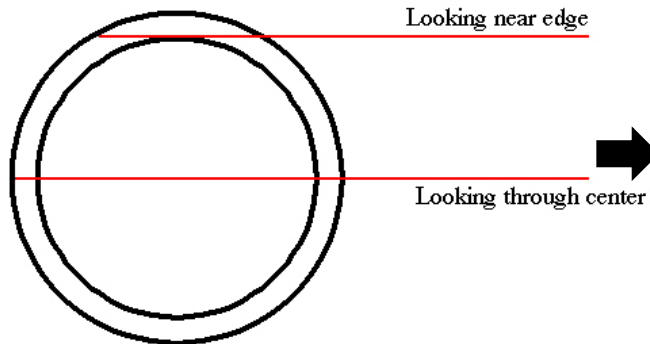
# Fermi Bubbles

Fermi data reveal giant gamma-ray bubbles

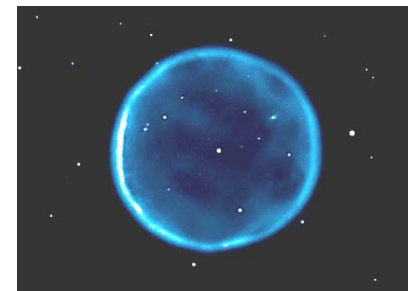


Credit: NASA/DOE/Fermi LAT/D. Finkbeiner et al.

Emission from thin shell

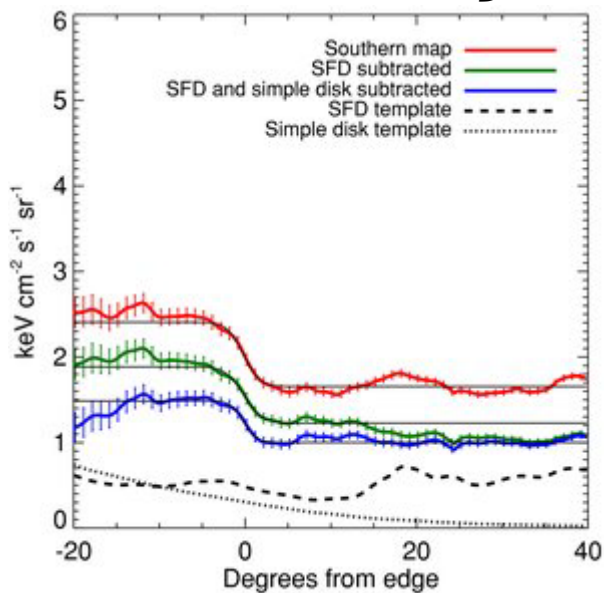


Surface brightness

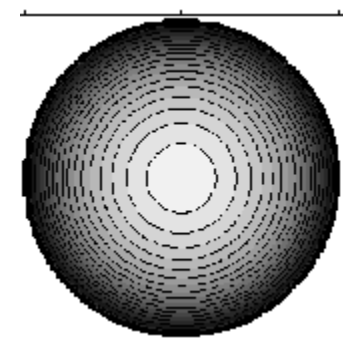
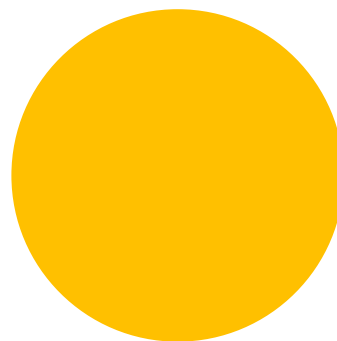


Limb Brightening

Uniform Surface brightness



Uniform Emissivity



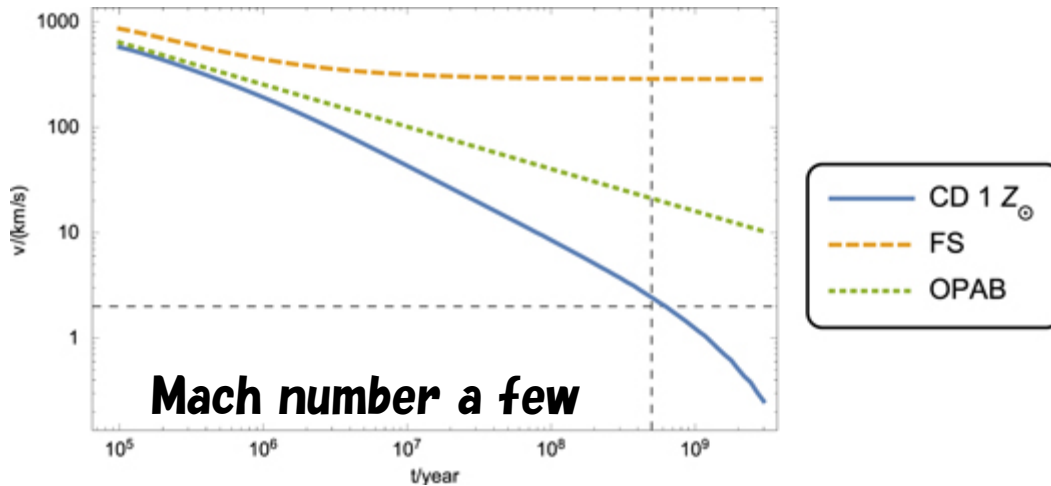
Center Brightening



indicates thick shell >> Electron cooling timescale  
 × shock speed (90pc for TeV e)

# Hadronic?

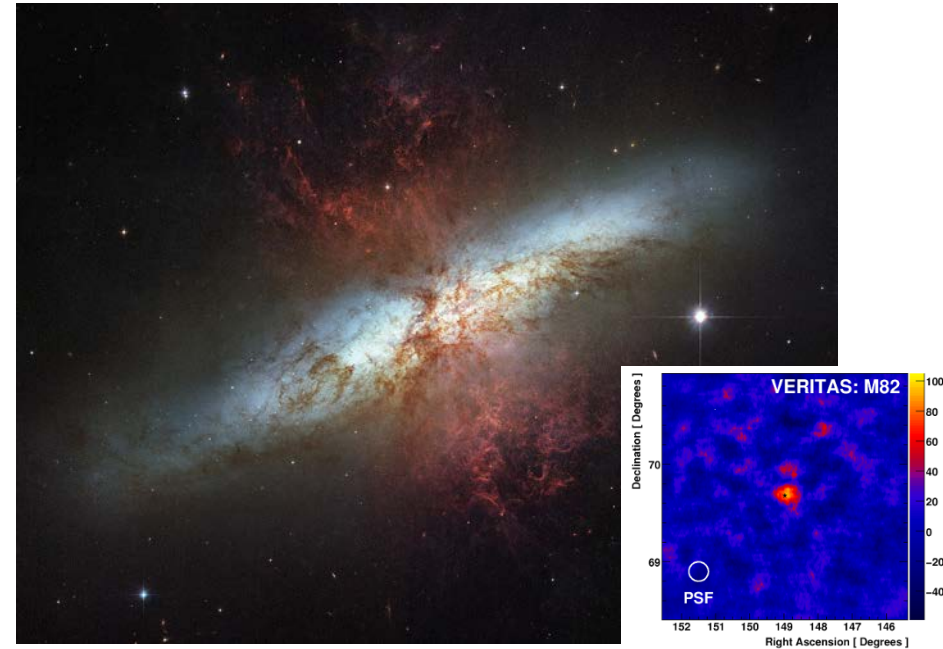
## Shock velocity (Crocker+ 2015)



$$10^{40} \text{ erg s}^{-1} \rightarrow 3 \times 10^{55} \text{ erg@}10^8 \text{ yr}$$

**Hadronic model requires  $> 10^{57}$  erg.**  
(CRs in the Galactic Disk:  $\sim 6 \times 10^{54}$  erg)

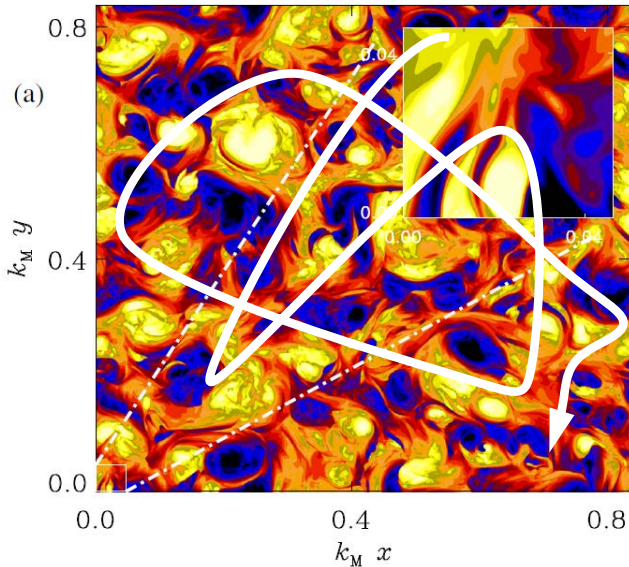
## M82 starburst galaxy ( $L \sim 3 \times 10^{42}$ erg/s)



**If the past GC activity deposited an energy  $> 10^{57}$  erg as the hadronic model implies, the structure may be more prominent as M82 shows. In a sense, the Fermi bubble is shabby or faint.**

**Weak shock + Thick Shell + Leptonic  $\rightarrow$  continuous electron acceleration via turbulence**

# Alternative Model



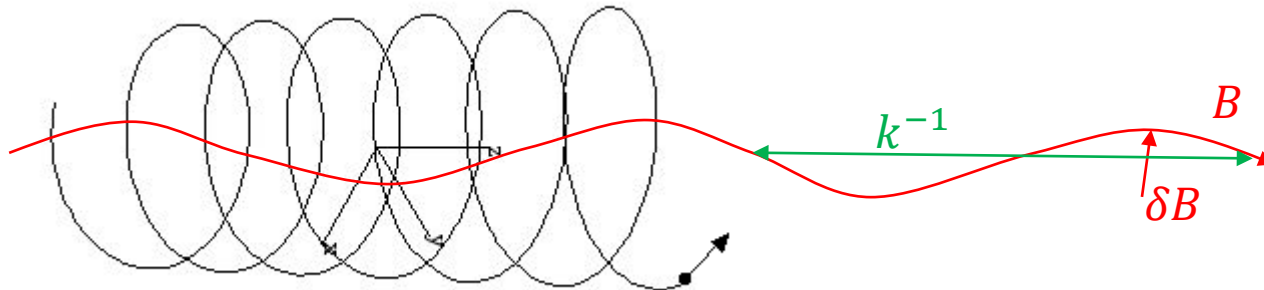
**Continuous Acceleration by Scattering with Turbulence. (2<sup>nd</sup> Order Fermi Acceleration)**

See e.g. Stawarz & Petrosian 2008

**Energy gain per scattering**

$$\frac{\Delta E}{E} \equiv \bar{\xi} \cong \frac{4}{3} \beta^2$$

**Alfvénic Wave (transverse/incompressible)**



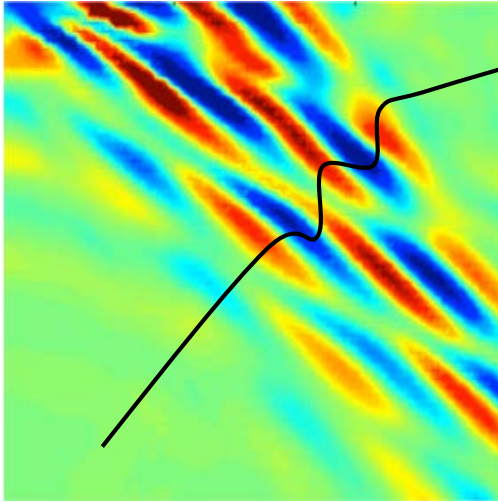
**pitch angle diffusion**  $\rightarrow$  **mean free path**  $l \sim \frac{B^2}{k \delta B^2(k)} r_L, k \sim 1/r_L$

**resonance condition**

$$\delta B^2(k) \propto k^{-q} \rightarrow D_{EE} = \frac{\langle \Delta E^2 \rangle}{\Delta t} \sim \frac{\bar{\xi} E^2}{l/c} \propto E^q$$

# Compressible wave

## Acoustic Wave (longitudinal/compressible)



### Mirror Force

$$\frac{\Delta p}{\Delta t} \sim \frac{p_{\perp} v_{\perp}}{2B} |\nabla B| \sim \frac{p_{\perp} v_{\perp}}{2B} k \delta B(k)$$

$$D_{EE} \sim \frac{c^2 \langle \Delta p^2 \rangle}{\Delta t} \sim \frac{E^2 c^2}{8B^2} \int d^3 k k_{\parallel}^2 \delta B^2(k) \frac{1}{k_{\parallel} v_{\text{ph}}}$$

**For fast wave with a typical eddy size  $L$**

$$D_{EE} \sim E^2 \frac{v_{\text{ph}}^2}{cL} \frac{\delta B_F^2}{B^2} \int_{k_{\text{min}}}^{k_{\text{max}}} d(Lk) (Lk)^{1-q} \sim E^2 \left( \frac{v_{\text{ph}}}{c} \right)^2 \frac{c}{L} \propto E^2$$

$$B^2 \uparrow \sim \delta B_F^2$$

$$t_{\text{acc}} \propto E^0$$

**(Hard sphere)**

**Ptuskin 1988: Cho & Lazarian 2006:  
Yan & Lazarian 2008: Lynn et al. 2014**



# Focker-Planck Equation

$$D_{EE} = KE^q$$

**Kolmogorov+Alfvenic  $q=5/3$ , Compressible  $q=2$  (hard sphere)**

$$\frac{\partial N_e(\varepsilon, t)}{\partial t} = \frac{\partial}{\partial E} \left[ D_{EE} \frac{\partial N_e(E, t)}{\partial E} \right] - \frac{\partial}{\partial E} \left[ \left( \frac{2D_{EE}}{E} - \langle \dot{E}_{\text{cool}} \rangle \right) N_e(E, t) \right] + \dot{N}_{e,\text{inj}}(E, t)$$

**Diffusion**

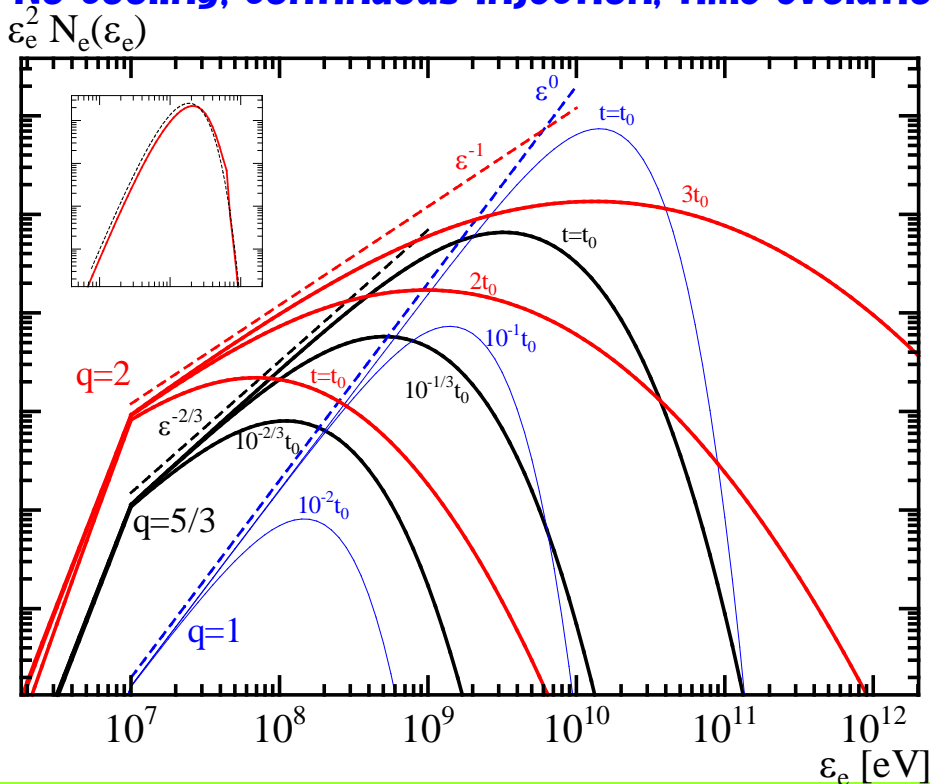
**Acceleration**

**Cooling**

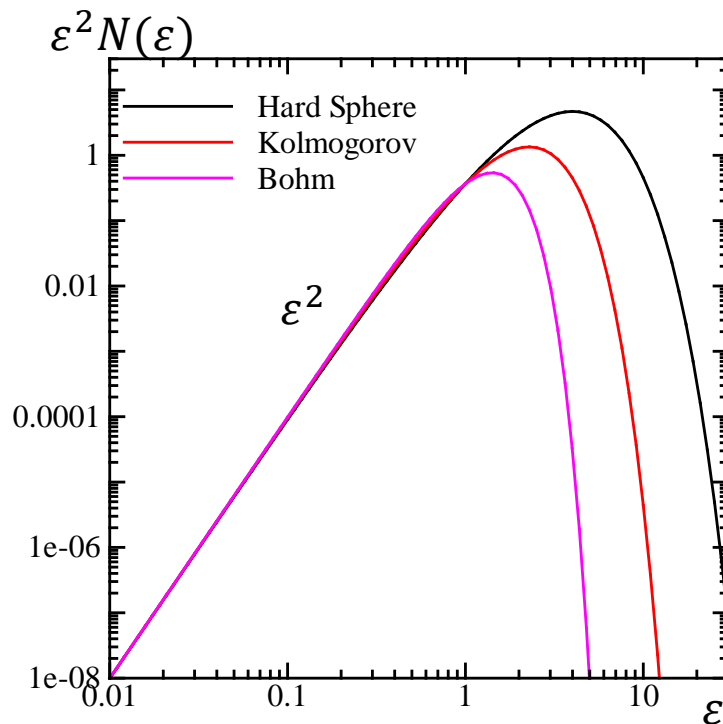
**Injection**

$$N(\varepsilon) \propto \varepsilon^{-1 \text{ or } -\frac{2}{3} \text{ or } 2} \quad \text{harder than the shock case} \quad N(\varepsilon) \propto \varepsilon^{-2}$$

**No cooling, continuous injection, time evolution**



**No injection, balance with cooling, steady**



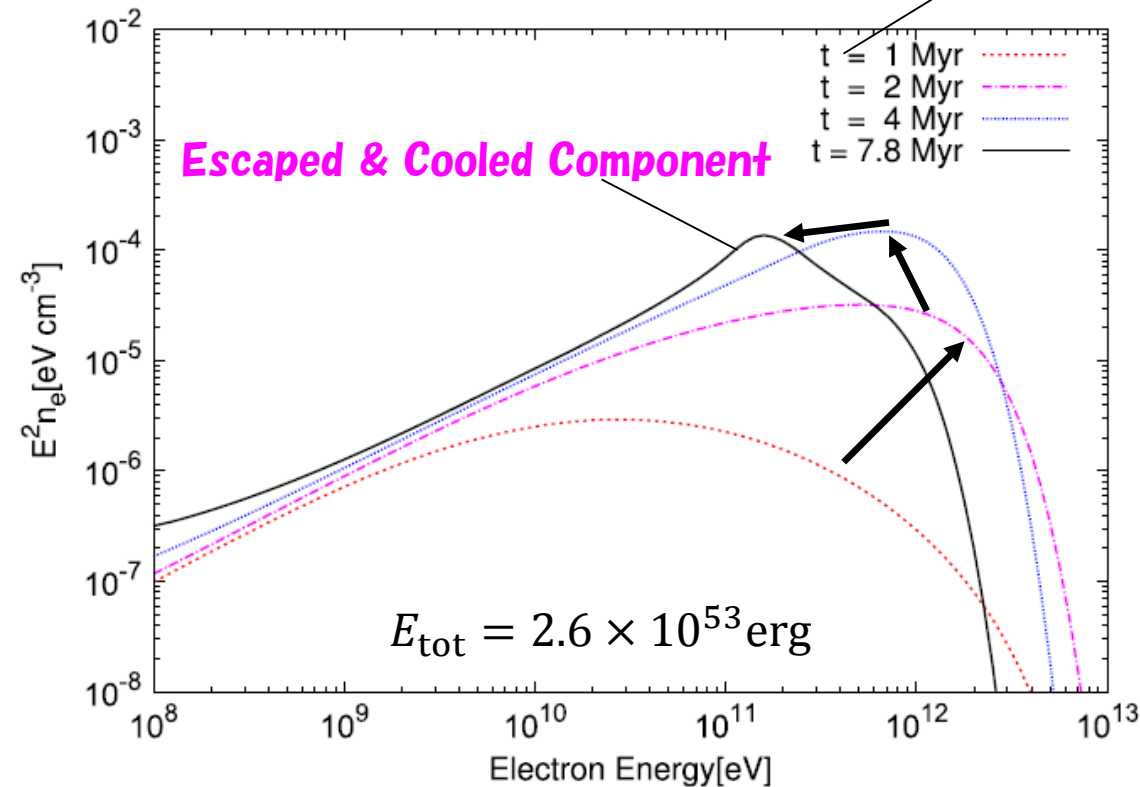
# Fermi Bubble

Sasaki, Asano & Terasawa 2015

$q = 2$  with escape effect from the disturbed region

Evolution of Electron spectrum

Elapsed time since the shock passed



Decaying turbulence

$$D(t) \equiv D_0 \left( 1 + \frac{t}{t_0} \right)^{-\alpha}$$

Spatial diffusion coefficient

$$D_{xx}(t) \equiv \frac{c^2}{3D}$$

Growing escape efficiency

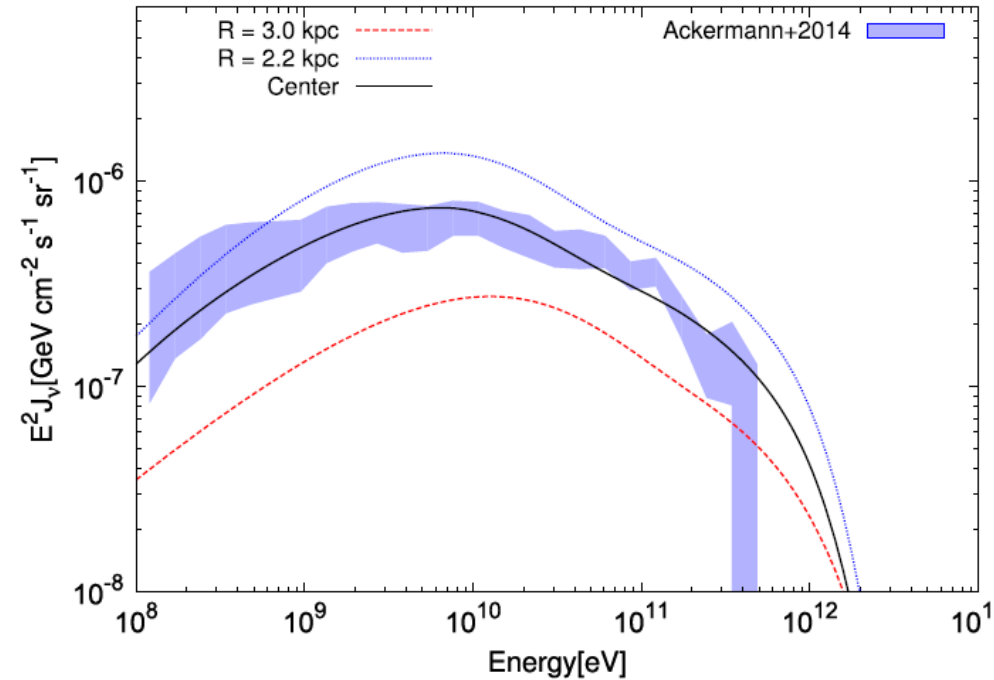
$$t_{\text{esc}} \equiv L^2 / D_{xx}$$

L: Region size  $\sim 180 \text{ pc}$

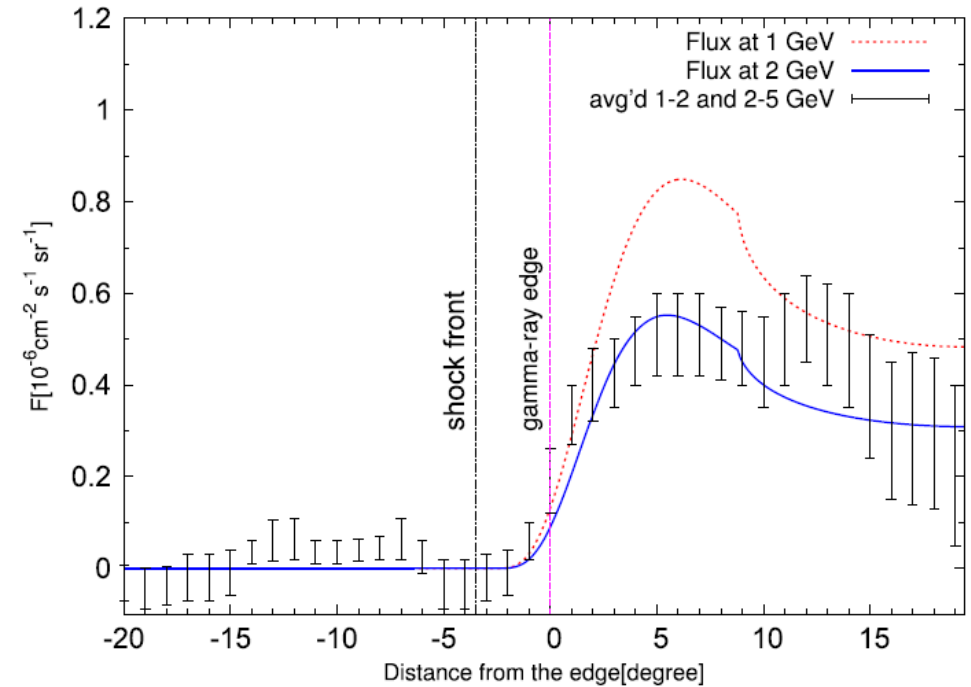
Slow process so the temporal evolution is essential.

# Fermi bubble

## Gamma-ray spectrum



## Surface brightness

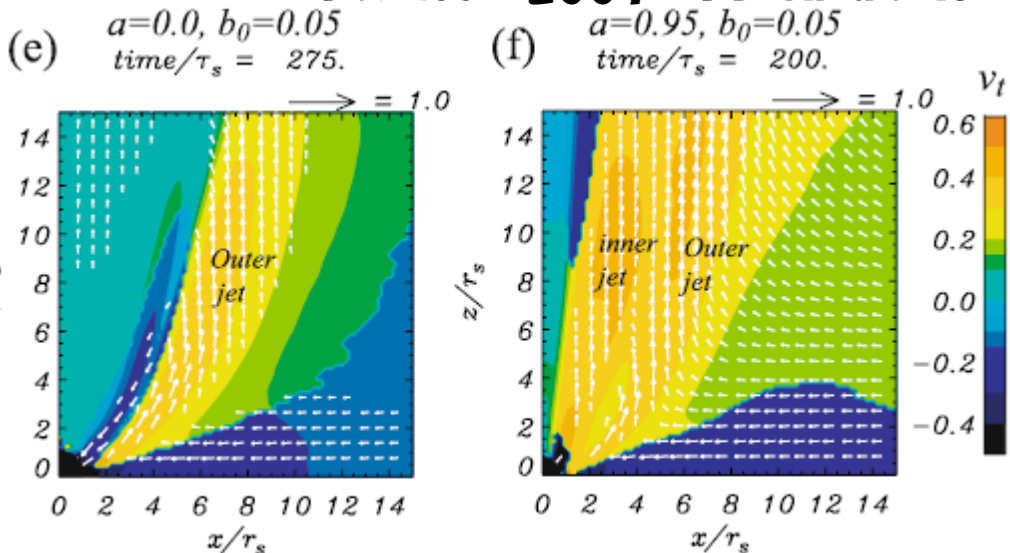


**The spectrum and surface brightness are reproduced by the phenomenological stochastic acceleration model.**

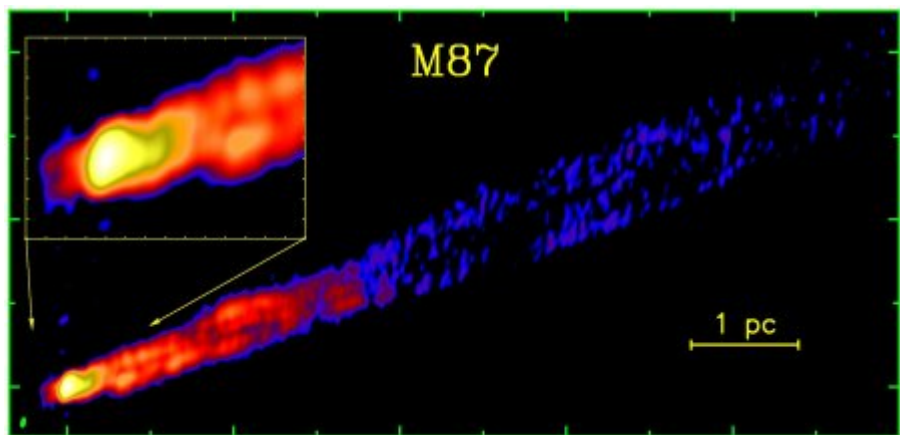
# Turbulence in AGN jets

## Spine-Sheath structure

Hardee+ 2007 MHD simulation



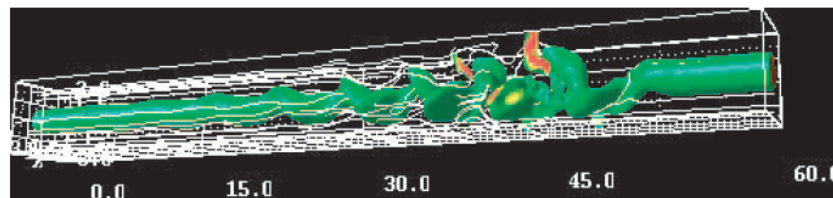
## Double layer in M87 jet



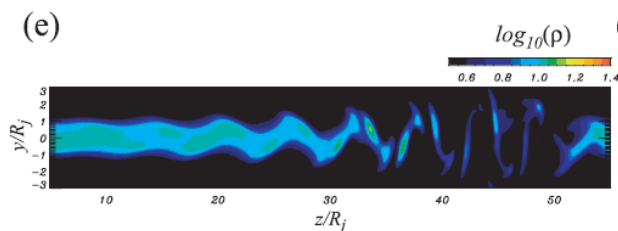
## Kelvin Helmholtz Instability

Mizuno+ 2007

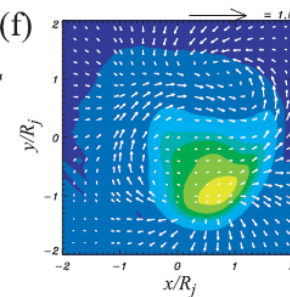
(d) RHD, wind,  $\omega=0.93$ , time=60.0



(e)



(f)



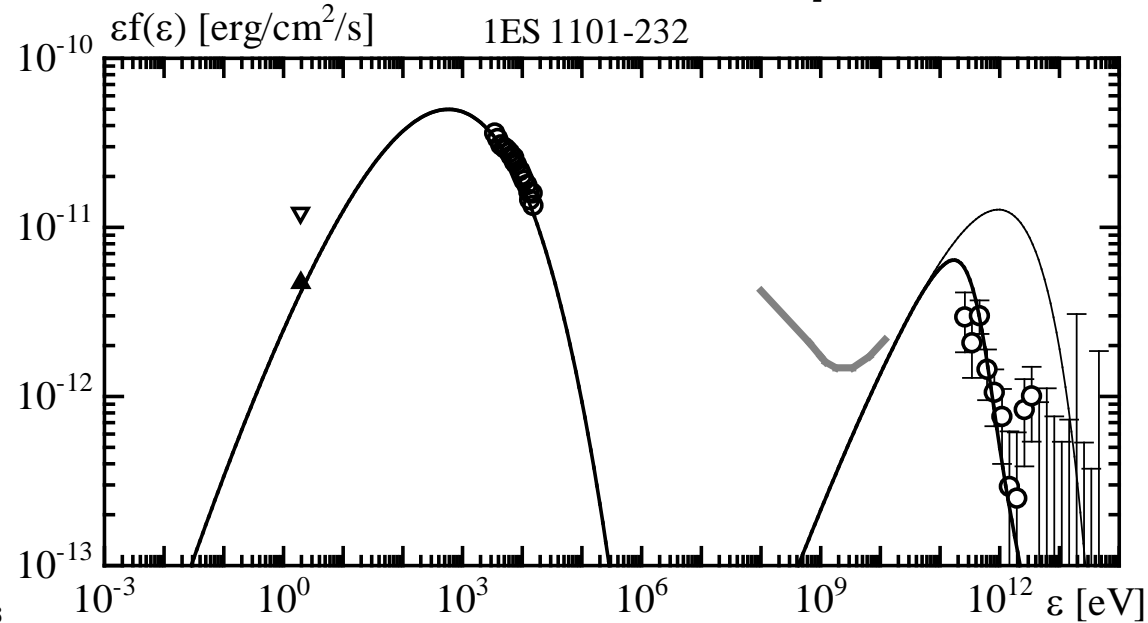
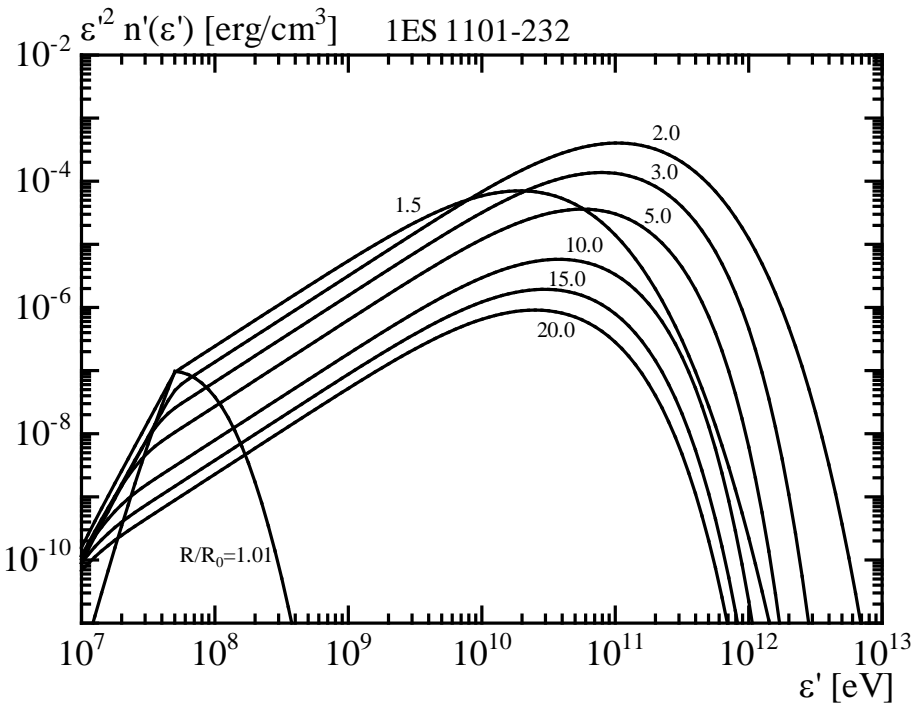
# Extreme Hard Blazar 1ES 1101-232

Asano+ 2014

$$L_\gamma = 2.6 \times 10^{43} \text{ erg s}^{-1}$$

Electron spectrum

Photon spectrum



The model parameters:  $\Gamma = 25$ ,  $B_0 = 0.03 \text{ G}$ ,  $W' = R_0/\Gamma = 2.8 \times 10^{16} \text{ cm}$ ,  $\Delta T'_{\text{inj}} = W'/c$ ,  
 $K = 4.3 \times 10^{-3} \text{ eV}^{1/3} \text{ s}^{-1}$ ,  $\dot{N}_0 = 1.5 \times 10^{46} \text{ s}^{-1}$

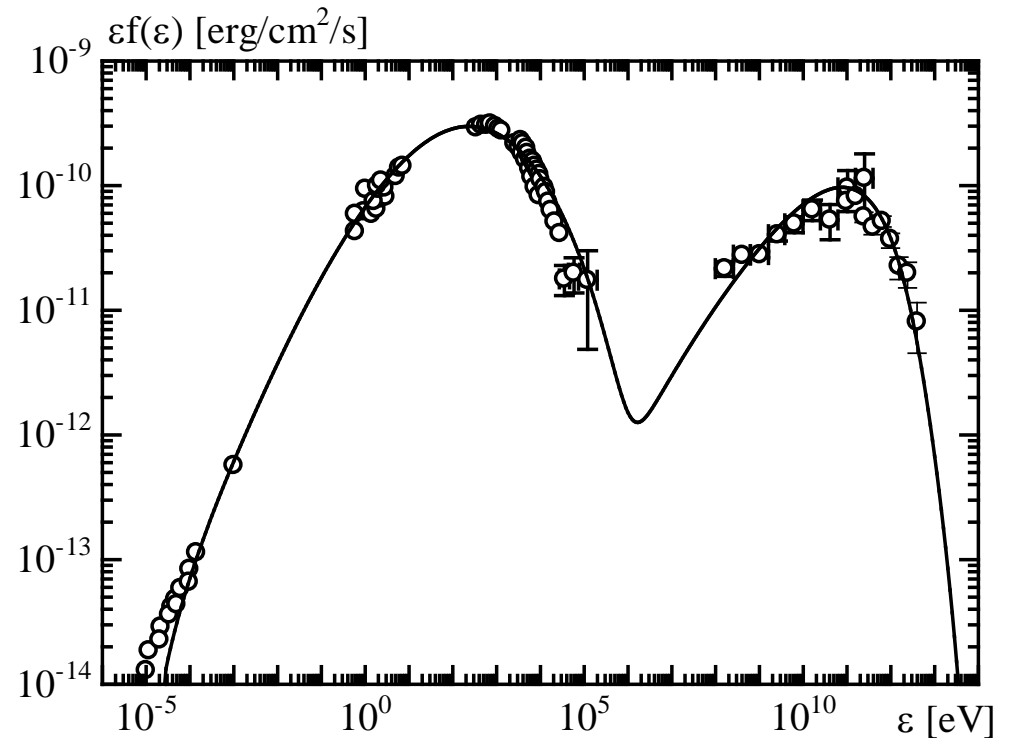
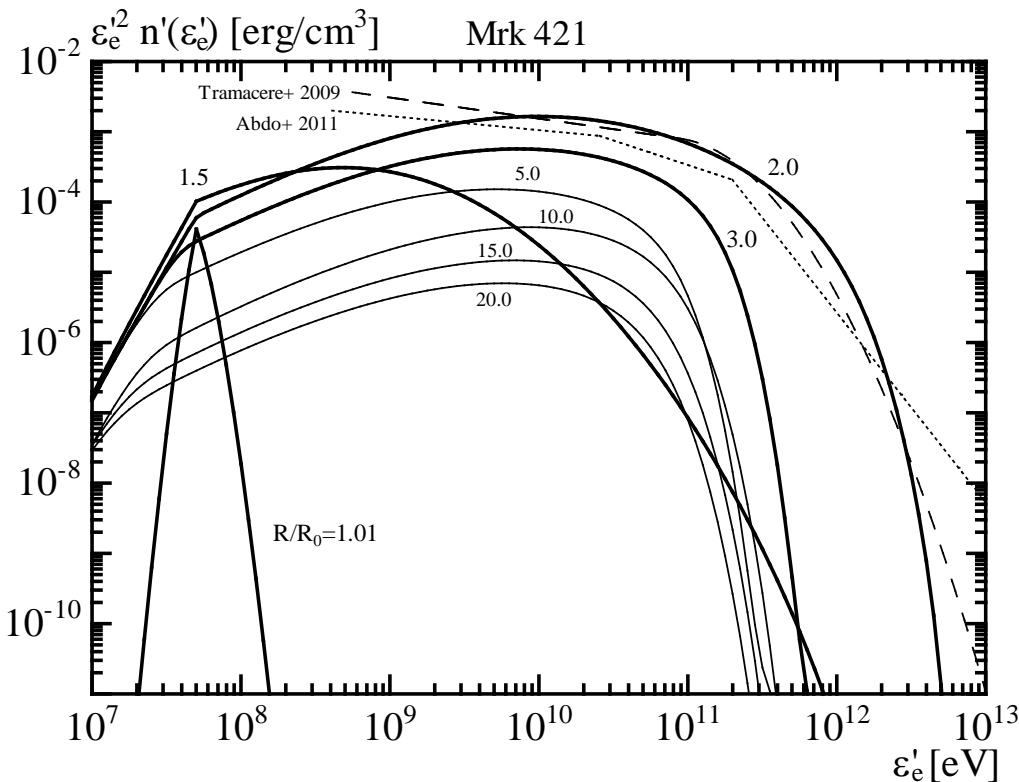
**Kolmogorov value  $q=5/3$**

**Expanding jet  $\rightarrow$  No steady state  $\rightarrow$  Temporal evolution is essential**

# Mrk421

$$q = 2$$

**Low maximum energy and curved electron spectrum are naturally reproduced with temporal evolution effects.**



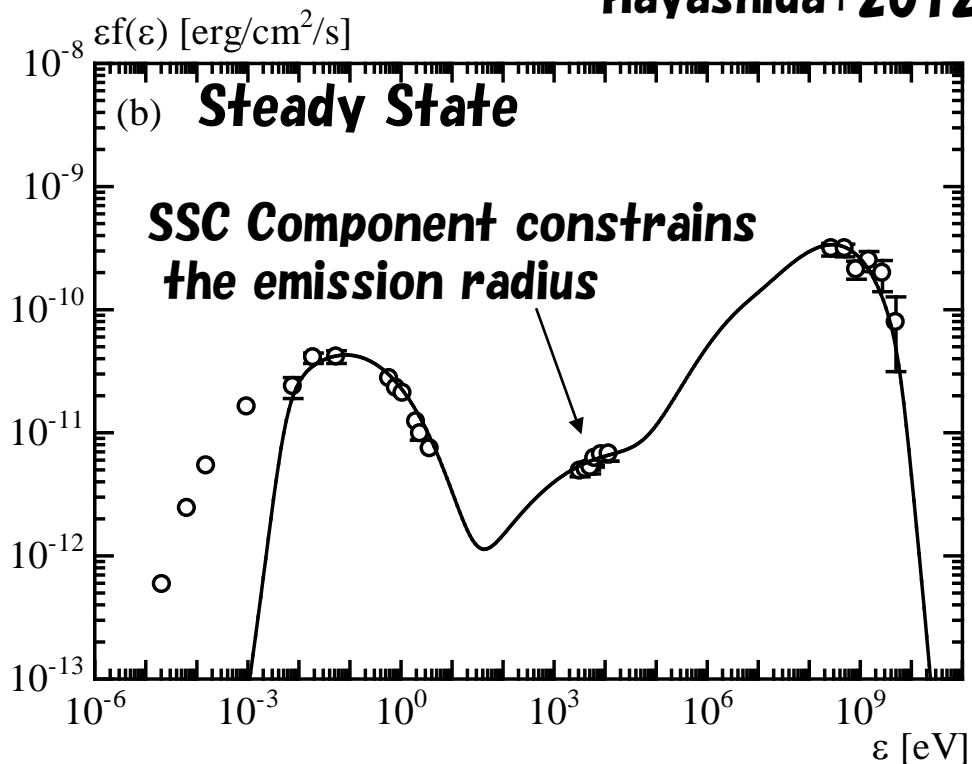
$$\Gamma = 15, B_0 = 0.16G, W' = \frac{R_0}{\Gamma} = 10^{16} \text{cm}, \Delta T'_{inj} = \frac{W'}{c}, K = 3.7 \times 10^{-6} \text{s}^{-1}, \dot{N} = 9.8 \times 10^{46} \text{s}^{-1}$$

**See also Kakuwa+ 2015**

# FSRQ 3C 279

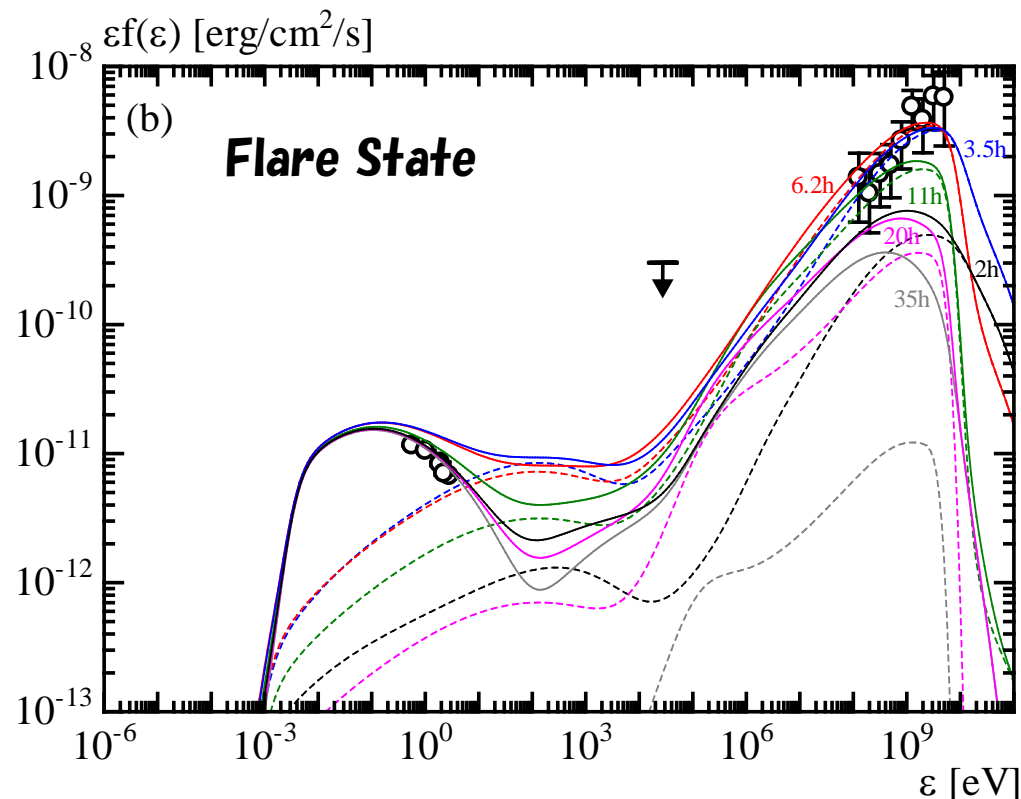
Asano & Hayashida 2015  
Hayashida+2012

$$T'_{UV} = 10\Gamma\text{eV}, U'_{UV} = 8 \left(\frac{\Gamma}{15}\right)^2 \text{erg cm}^{-3}$$



$$q = 2$$

parameters are  $R_0 = 0.023 \text{ pc}$ ,  $\Gamma = 15$ ,  $K' = 9 \times 10^{-6} \text{ s}^{-1}$   
 $(t_{\text{acc}} = 1/(2 K') = 0.35W'/c)$ ,  $\dot{N}'_e = 7.8 \times 10^{49} \text{ s}^{-1}$  ( $\dot{n}'_e =$   
 $0.26(R/R_0)^{-2} \text{ cm}^{-3} \text{ s}^{-1}$ ), and  $B_0 = 7 \text{ G}$ .



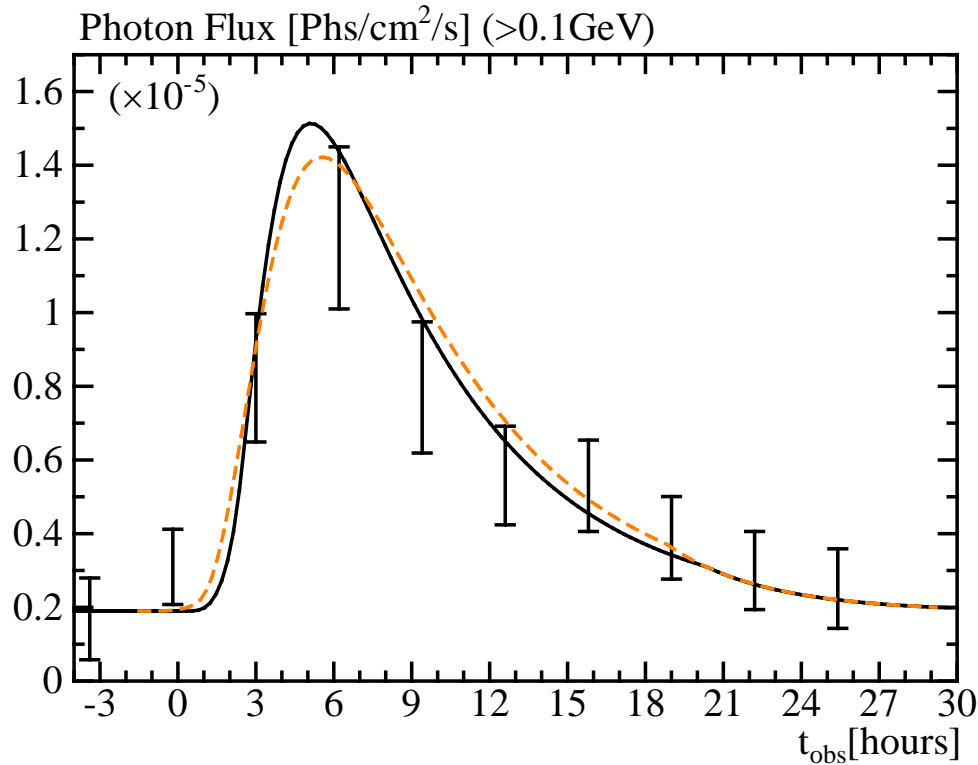
$$K' = 1.3 \times 10^{-5} \text{ s}^{-1} \quad (t_{\text{acc}} = 1/(2 K') = 0.25W'/c),$$

$$\dot{N}'_e = 2.5 \times 10^{50} \text{ s}^{-1} \quad (\dot{n}'_e = 0.85(R/R_0)^{-2} \text{ cm}^{-3} \text{ s}^{-1}),$$

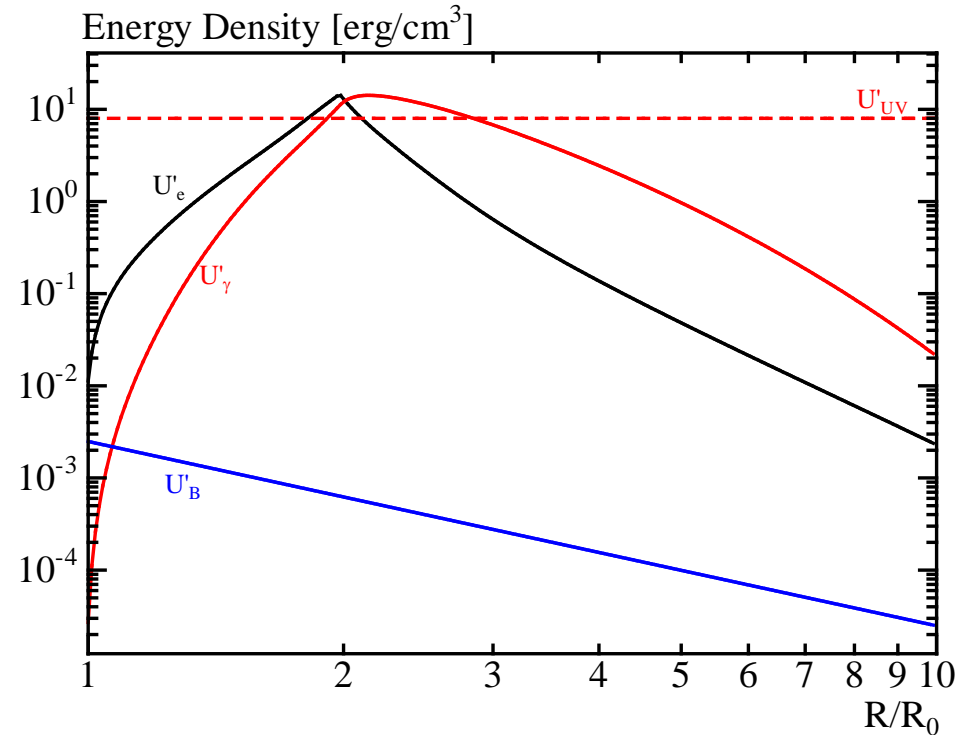
$$B_0 = 0.25 \text{ G}.$$

# 3C 279 Flare

## Lightcurve



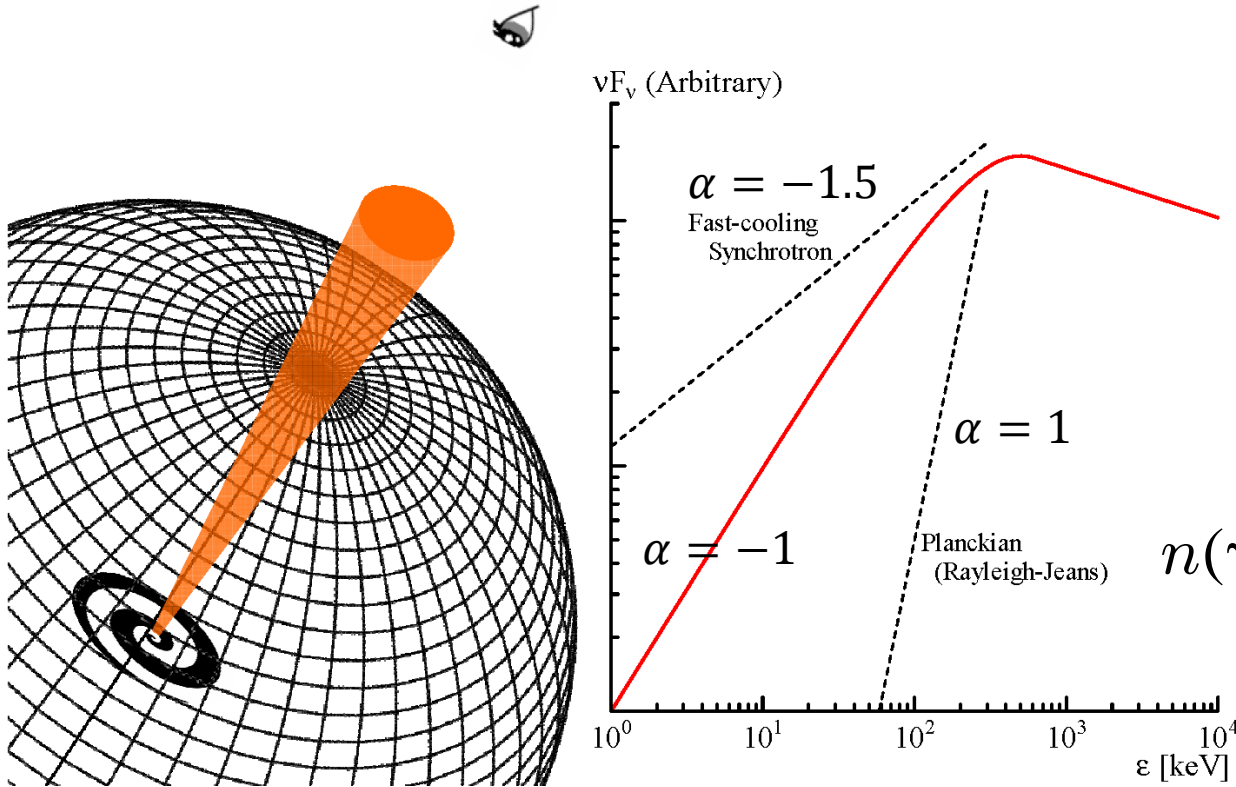
## Evolution of energy density



**Weak magnetic field:  
Acoustic waves rather than Reconnection**



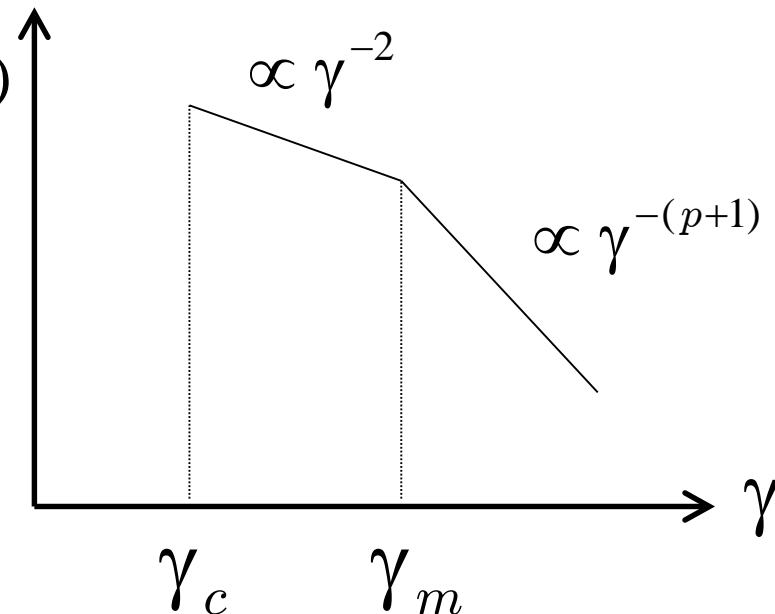
# Gamma Ray Burst



**Softer than the thermal spectrum,  
but harder than the synchrotron spectrum.**

**See Daigne, Bosnjak, Dubus 2011**

- **Power-law injection with a minimum energy leads to an electron spectrum below.**
- **The index for cooled electrons becomes 2, then photons index should be 1.5.**

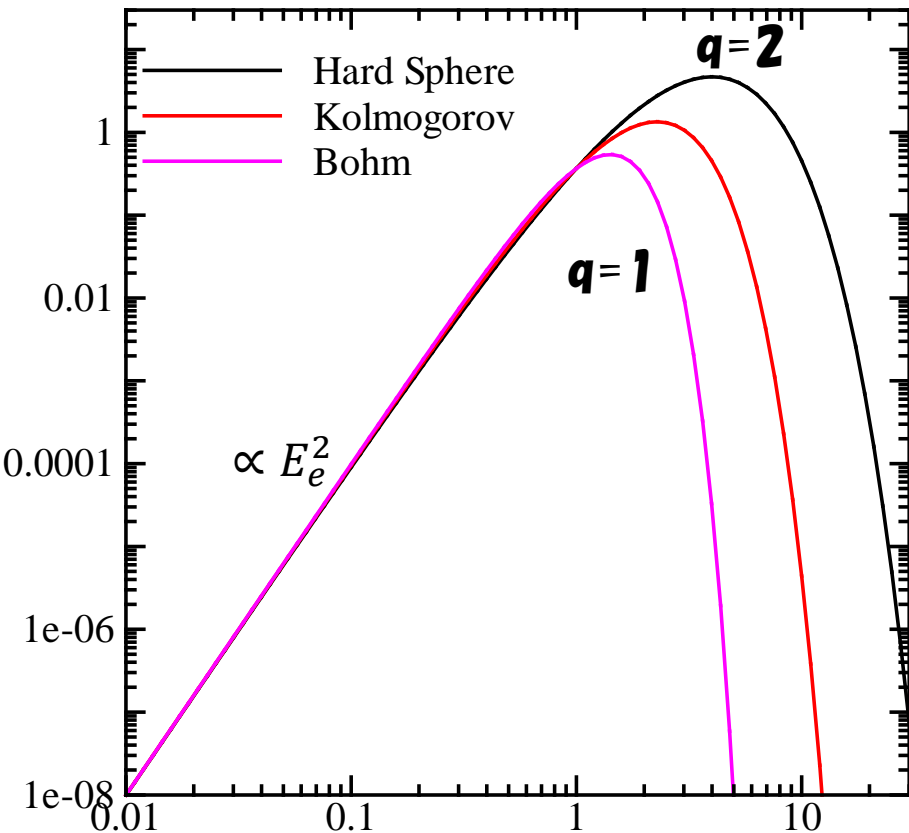


# Balance between Acceleration and Cooling

Magnetic Field is so strong that balance with acceleration.

Analytic Steady Solution for Electrons

$$D(\varepsilon) = \frac{\bar{\xi} \pi e c \varepsilon k \delta B_k^2}{8B} \equiv K \varepsilon^q.$$

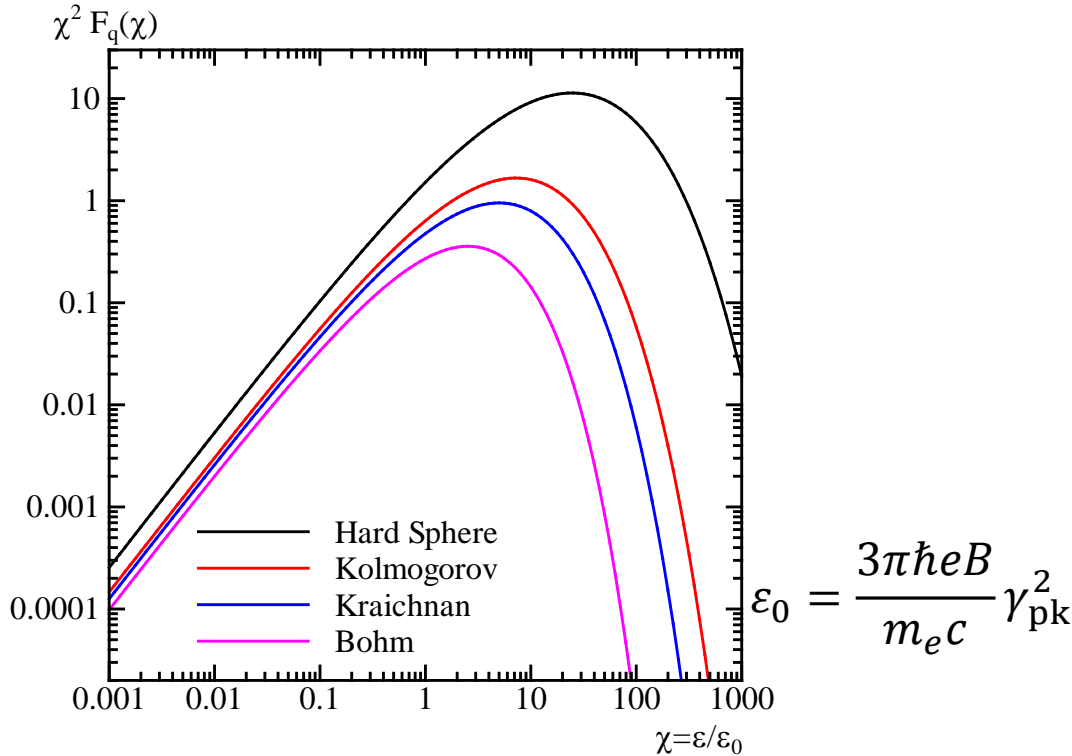


$$N(E_e) = \frac{3N_{\text{tot}}}{\Gamma\left(\frac{6-q}{3-q}\right) E_{\text{pk}}} \left(\frac{E_e}{E_{\text{pk}}}\right)^2 \exp\left[-\left(\frac{E_e}{E_{\text{pk}}}\right)^{3-q}\right],$$

$$E_{\text{pk}} = \left(\frac{6\pi(3-q)K_0 m_e^2 c^3}{\sigma_T B^2}\right)^{1/(3-q)}.$$

See Lefa+ 2011

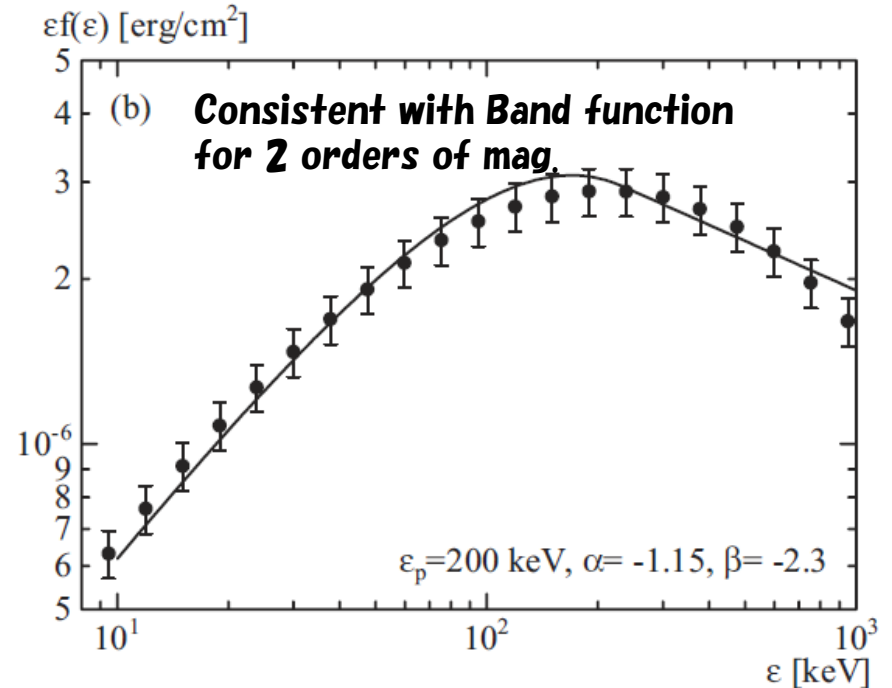
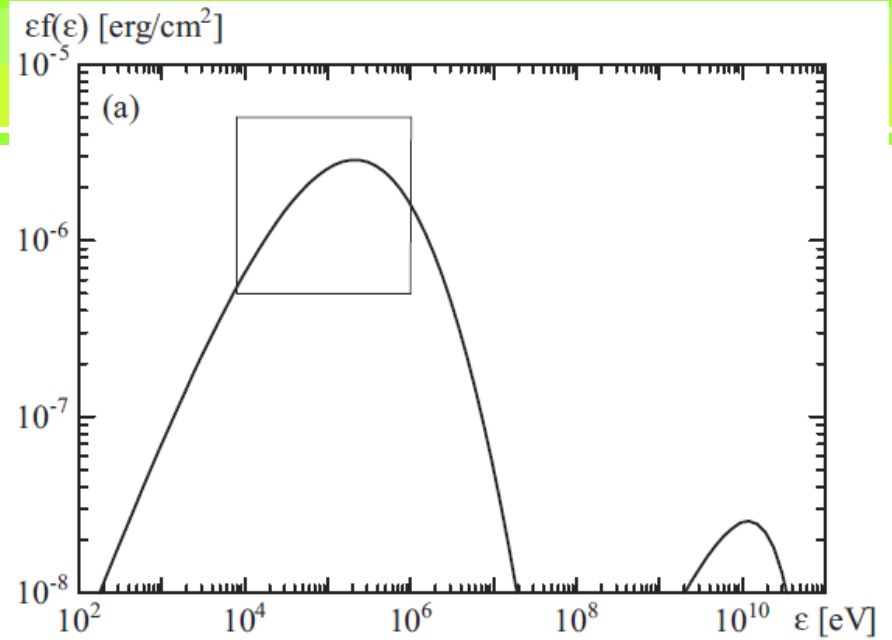
# Synchrotron Photon



$$\varepsilon_0 = \frac{3\pi\hbar e B}{m_e c} \gamma_{pk}^2$$

$$\varepsilon_p = 1.0 \left( \frac{\Gamma}{500} \right) \left( \frac{K_0}{10^2 \text{ s}^{-1}} \right)^2 \left( \frac{B}{10^4 \text{ G}} \right)^{-3} \text{ MeV, for } q = 2,$$

$$l_{\text{edd}} \sim ct_{\text{acc}} \beta_{\text{eff}}^2 \sim 3 \times 10^7 (t_{\text{acc}}/10^{-2} \text{ s}) (\beta_{\text{eff}}^2/0.1) \text{ cm}$$

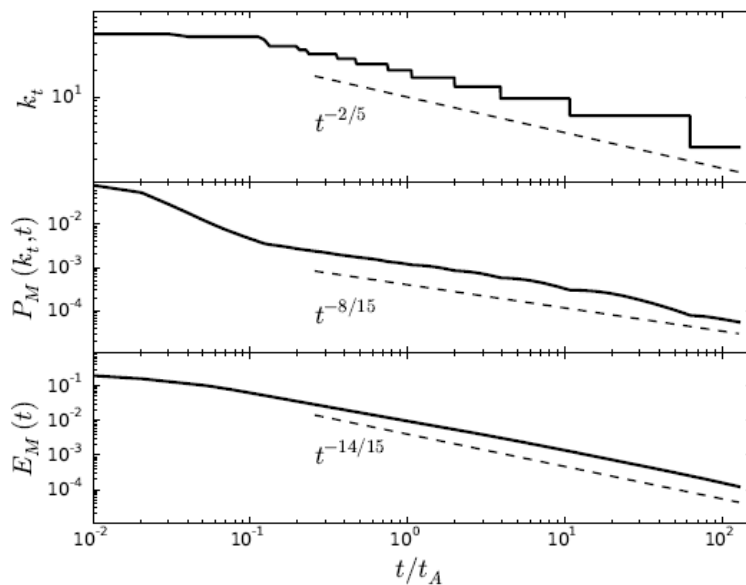
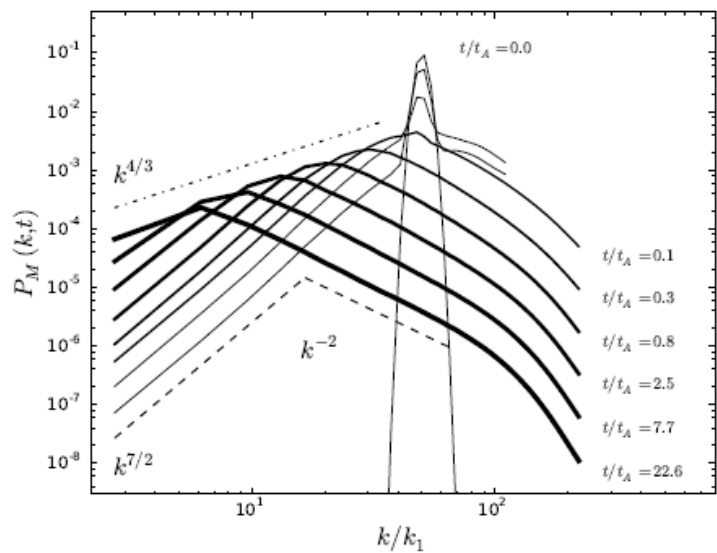
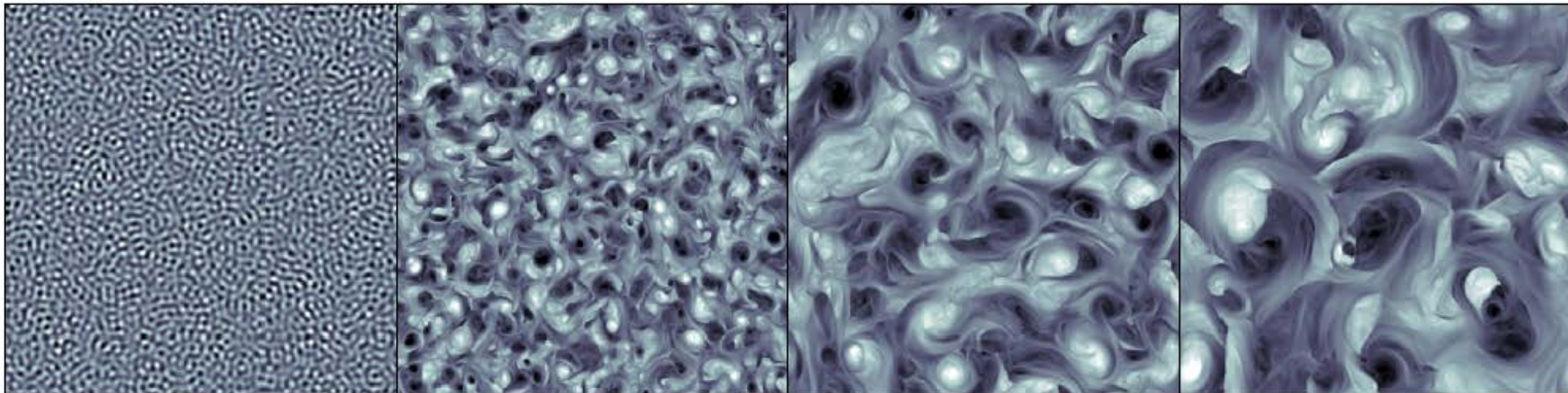


# Decaying Turbulence

INVERSE CASCADE OF NON-HELICAL MAGNETIC TURBULENCE IN A RELATIVISTIC FLUID

Zrake 2014

JONATHAN ZRAKE



# Temporal Evolution of the Spectrum

$$\begin{aligned}
 N_\gamma(\varepsilon') &= \int dt \dot{N}_\gamma(\varepsilon') \\
 &= \frac{\sqrt{3}e^2 m_e^2 c^3}{\pi \Gamma\left(\frac{6-q}{3-q}\right) \hbar^2} \int dt \frac{N_{\text{tot}}}{E_{\text{pk}}^2} F_q\left(\frac{\varepsilon'}{\varepsilon'_0}\right)
 \end{aligned}$$

**Electron Number: Increase with injection**  
**Electron peak energy: Decay as the efficiency of acceleration/cooling drops.**

**The temporal evolutions of two combinations determine the final photon spectrum**

$$N_{\text{tot}}/E_{\text{pk}}^2 \propto (t + t_0)^a \quad \text{and} \quad \varepsilon'_0 \propto B E_{\text{pk}}^2 \propto (t + t_0)^{-b}$$

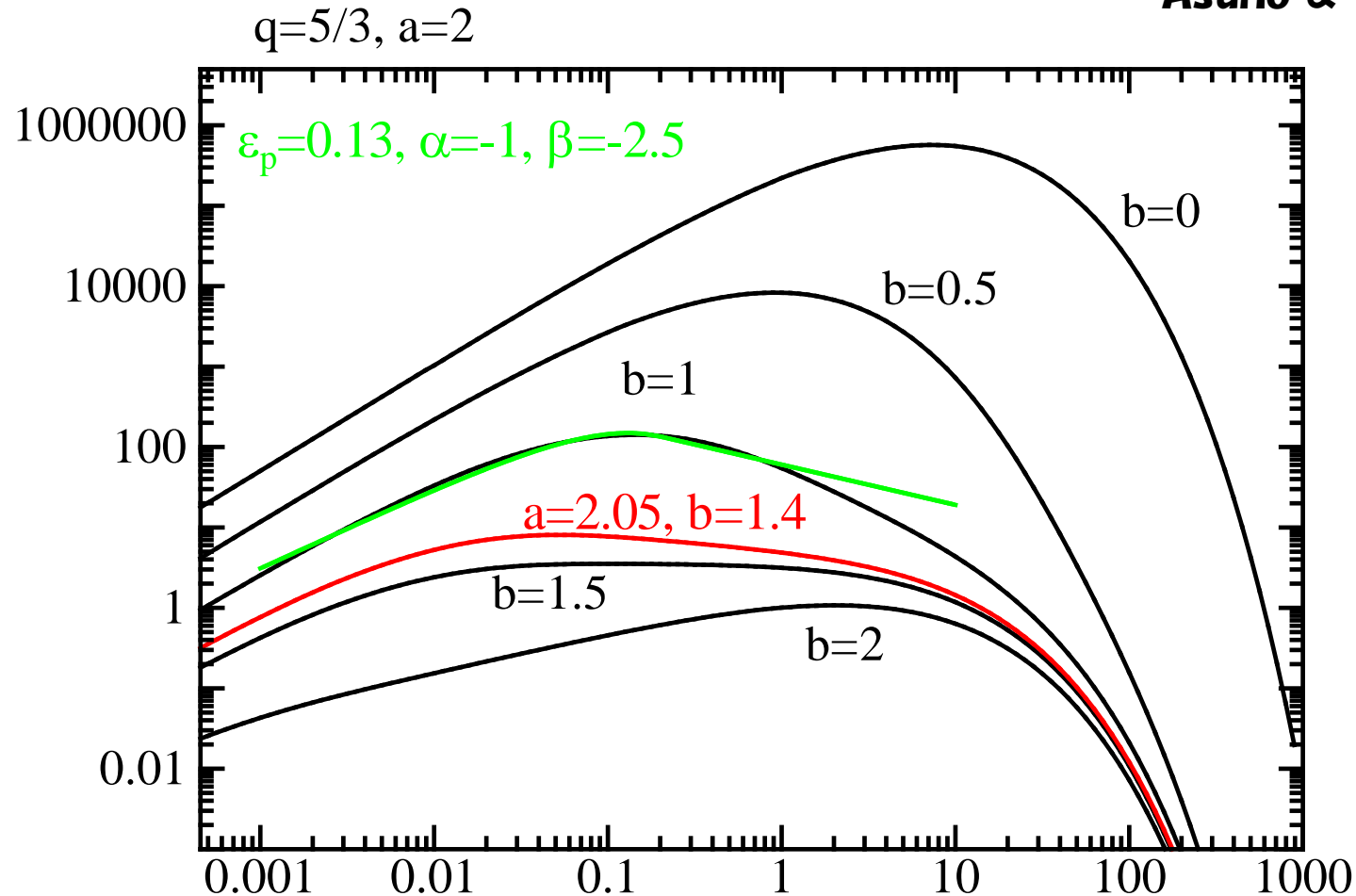
**High-energy photon index**  
 $\beta = -(1 + a)/b$

**Note**

$$E_{\text{pk}} = \left( \frac{6\pi(3-q)K_0 m_e^2 c^3}{\sigma_T B^2} \right)^{1/(3-q)}$$

# Band-like Spectra

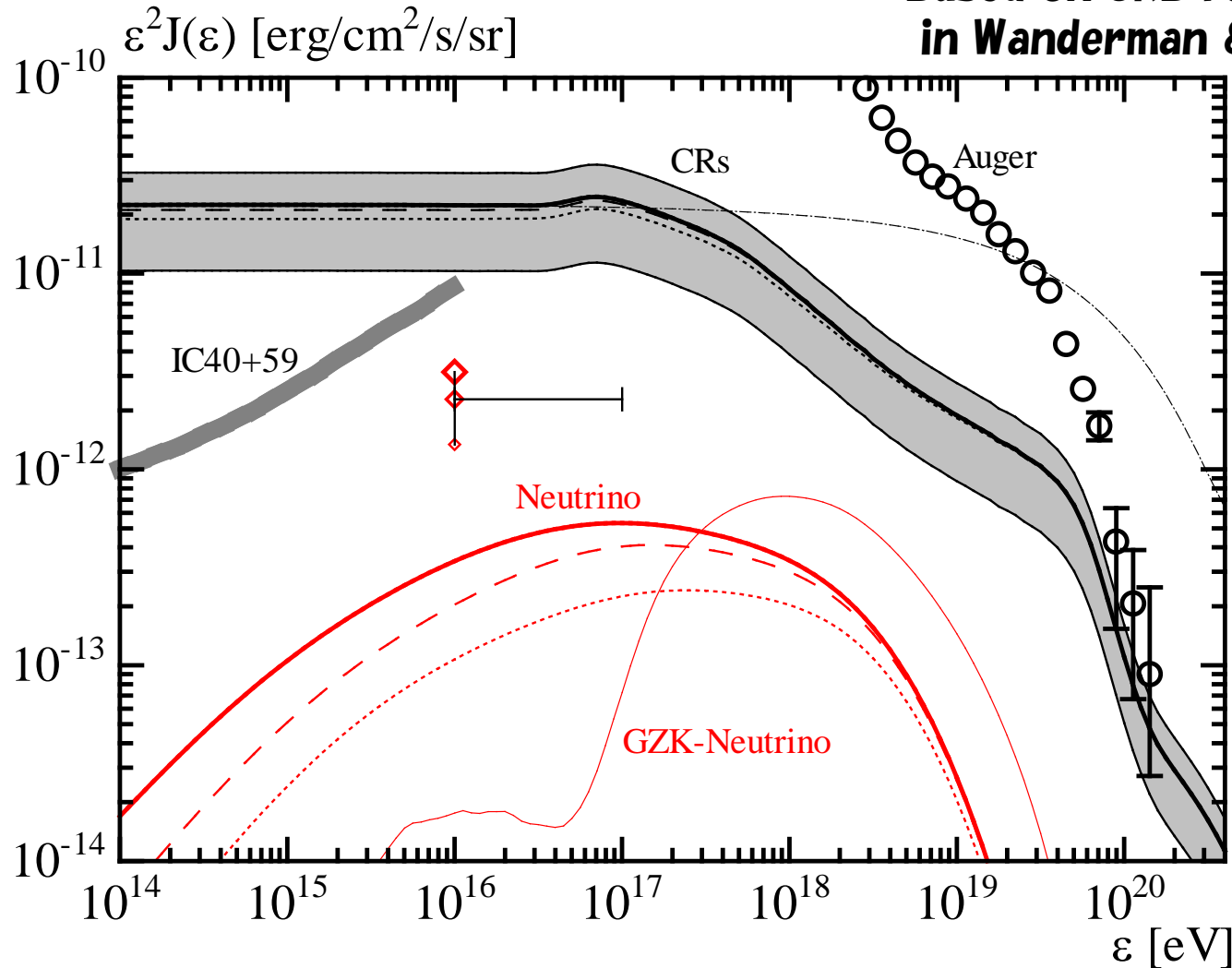
Asano & Tersawa 2015



**The required indices and MHD simulations seem to be reasonable.**

# Ultra High-Energy Cosmic Ray from GRBs

Based on GRB rate & Luminosity function in Wanderman & Piran 2010.



Assuming  $E^{-2}$ -spectrum,

$$\frac{L_p}{L_\gamma} = 10, \frac{L_B}{L_\gamma} = 0.1$$

In this shock acceleration model, only the highest energy region is explained.

Asano & Meszaros 2014

# Summary

- The stochastic acceleration is a slow process, but enough to explain the **Fermi bubble**.
- In relativistic jets (AGN, GRB...), the acceleration timescale can be comparable to the shock acceleration.
- The **curved spectra in blazars** are reproduced by the stochastic acceleration model with the temporal evolution effect.
- The problem in the **GRB spectra** is solved by the balance between the acceleration and cooling with decaying turbulence.
- The hard spectrum in this GRB model is favorable to agree with the **UHECR flux**.