宇宙の物質、磁場、背景輻射中で発達する電子光子シャワーの特徴

岡山商科大　　中塚隆郎

共同研究　　岡山理科大学、川崎医科大学、JAXA

Contents

- New important cascade studies for astrophysical research
- Analytical approaches, Monte Carlo approaches, and numerical approches
- Our numerical approach to solve astrophysical electromagnetic cascades
- Present stage of our evaluations
- Transition mechanism of differential energy spectrum for cascades in photon gas
- Conclusions

Classical cascade shower theories

- Landau L and Rumer G, 1938 *Proc. R. Soc.* **A 166** 213
- Rossi B and Greisen K 1941 *Rev. Mod. Phys.* **13** 240
- Nishimura J 1957 *Handbuch der Physik* vol XLV1/2, 1 (Berlin: Springer)

CASCADES in astrophysical environments, a new important application

- Akhiezer A I, Merenkov N P, and Rekalo A P

  Cascades in strong magnetic fields
- Anguelov V, and Vankov V

  Cascades in strong magnetic fields
- Aharonian F A, Kirillov-Ugryumov V G, and Vardanian V V

  Cascades in background photon fields
- Zdziarski A A

  Cascades in background photon fields
- Aharonian F A, and Plyasheshnikov A V

  Cascades in matter, photon, and magnetic fields

Analytical approaches, Monte Carlo approaches, and numerical approaches

Akhiezers' analytical result in adiabatic approximation (AA)

is denied by

Anguelovs' Monte Carlo result (MC);

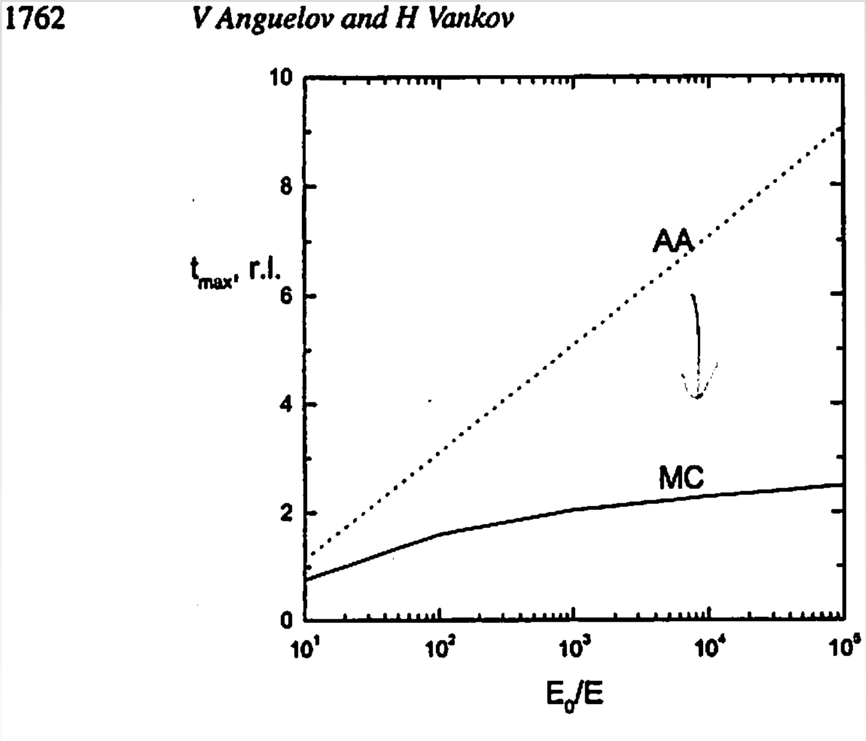


Figure 7. Depth of the maximum as a function of E_0/E.

Aharonian and Plyasheshnikov say in *Astrop. Phys.* **19**, 525(2003);

> In the cases of magnetic field and photon gas, such a nice feature of cross-section dependences is lacking. As a result, the analytical solution of cascade equations becomes more complicated and does not provide an adequate accuracy.

So that some numerical approches are required.

Aharonian and Plyasheshnikov applied a traditional "adjoint method"

15th ICRC, I ('77), Plovdiv

GENERAL EQUATIONS FOR THE GENERATING FUNCTIONAL AND FACTORIAL MOMENTS IN THE CASCADE THEORY

UCHAIKIN V.V.

ALTAI STATE UNIVERSITU, BARNAUL, USSR.

The non-linear equation for the generating functional and factorial moments is obtained in general form. The linear equations for first and higher order factorial moments of random measure which describes the collision number of all particle generations in some area of the phase space can be derived from this equation. Application of these results to the cascade theory is discussed.

Our numerical approach

Main characteristic points of our method are

- The shower development with inhomogeneous cross-sections for primary and secondary energies, like Akhiezer et al's or Zdziarski's, are solved.
- We use logarithmic scale in energy.
- The calculation time are reduced to be proportional to $\ln E_0/E$ or $\ln^2 E_0/E$, much less than E_0/E in Monte Carlo methods.

Our result for Cascades in strong magnetic fields

Akhiezer et al equation is described as

$$\frac{d\Pi(\varepsilon,x)}{dx} = 2\int_{\varepsilon}^{\infty} \Gamma(u,x)\gamma(u,\varepsilon)\,du + \int_{\varepsilon}^{\infty} \Pi(u,x)\pi(u,u-\varepsilon)\,du - \int_{0}^{\varepsilon} \Pi(\varepsilon,x)\pi(\varepsilon,\varepsilon-u)\,du$$

$$\frac{d\Gamma(\varepsilon,x)}{dx} = \int_{\varepsilon}^{\infty} \Pi(u,x)\pi(u,\varepsilon)\,du - \int_{0}^{\varepsilon} \Gamma(\varepsilon,x)\gamma(\varepsilon,u)\,du$$

where

$$\pi(\varepsilon,\omega)\,d\omega = q\,\frac{[1+(1-y)^2]}{(1-y)^{1/3}y^{1/3}}\frac{dy}{\omega^{1/3}}$$

$$\gamma(\omega,\varepsilon)\,d\varepsilon = q\,\frac{[(1-x)^2+x^2]}{(1-x)^{1/3}}\frac{dx}{\varepsilon^{1/3}}.$$

Comparison of cascades in the strong magnetic fields

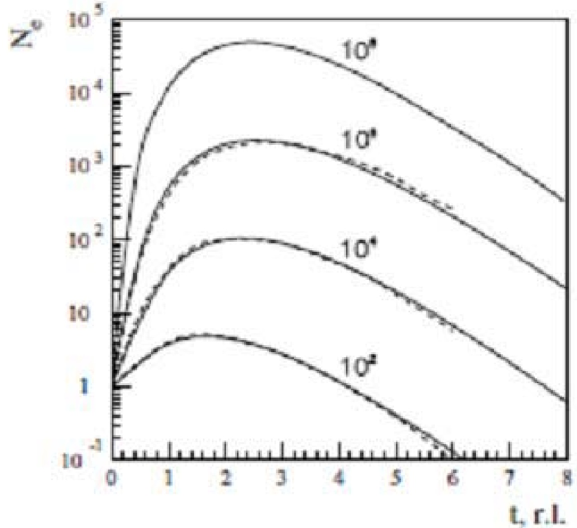


Fig. 11. Cascade curves of electrons for showers initiated by primary electrons in the magnetic field. Different values (indicated at the curves) of the ratio of primary and threshold energies are assumed for the fixed $\chi_{0k} = \varepsilon_{0k} H/H_{cr} = 10^3$. For comparison, the results obtained in Ref. [24] are also shown (dashed curves).

Figure 1: Aharonian-Plyasheshnikov's Fig. 11.

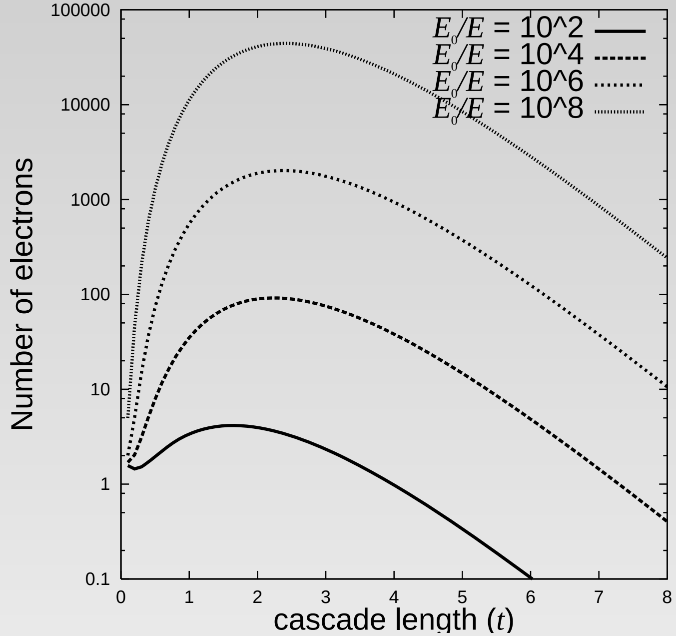
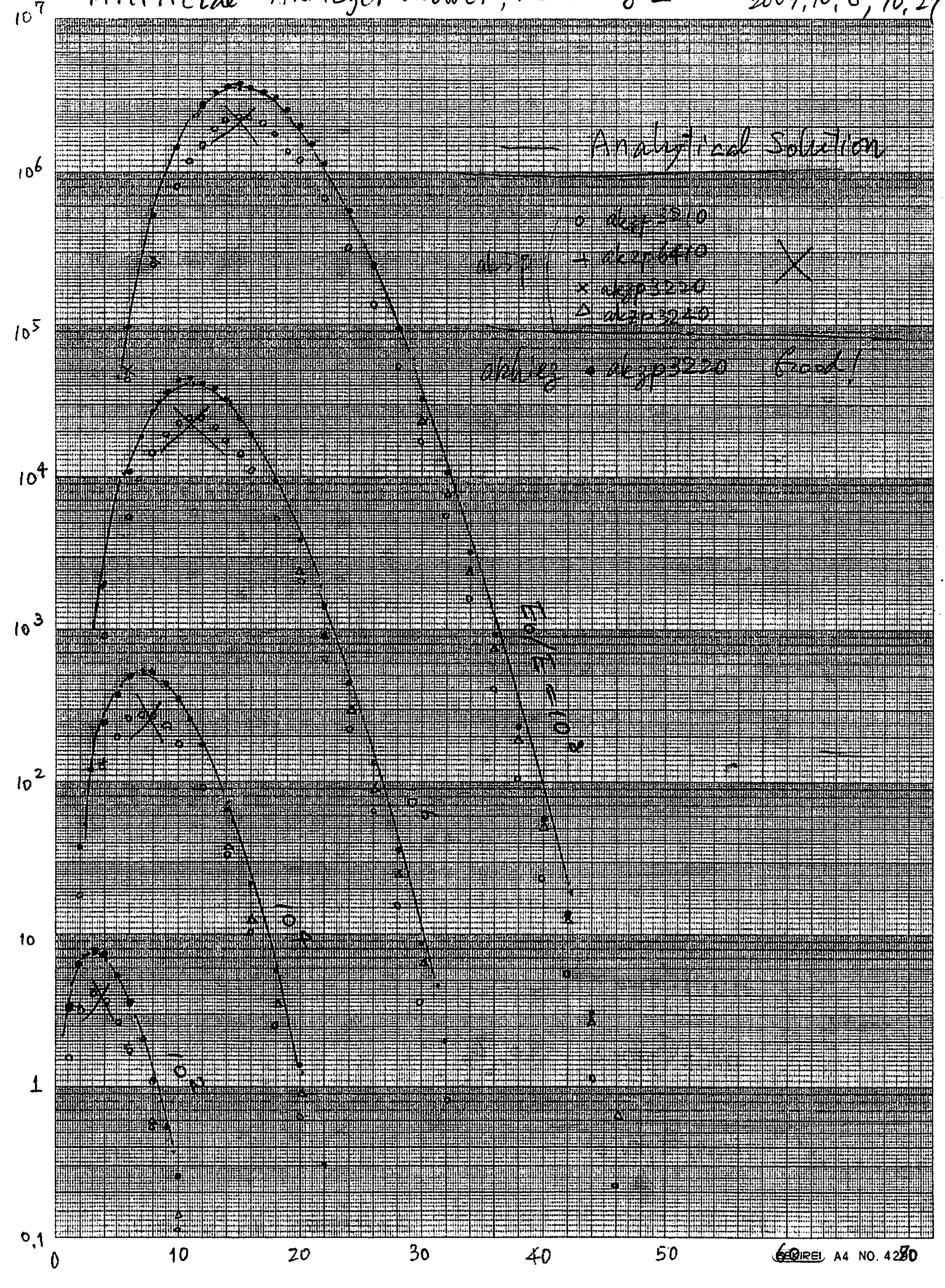


Figure 2: Our result

断面積の中の非同次の係数、 $E^{-1/3}$、を人為的に撤去すると、方程式は解析的に解ける。

この人為的なシャワーについて点検のため、われわれの数値的な手法で解いた結果と解析解を比較する。

Artificial Akhiezer Shower, Removing $E^{-1/3}$ 2009, 10, 3, 10, 27



Our cascade results in matter

The diffusion equation is described as

In the thickness dt the number of photons with energy between W and $W+dW$ undergoes a change because of the following effects:

(a) Electrons with energy E larger than W radiate a certain number of photons in the energy interval (W, dW). This number is

$$dW\,dt\int_W^\infty \pi(E,t)\varphi_0\left(\frac{W}{E}\right)\frac{dE}{E} = dW\,dt\int_0^1 \pi\left(\frac{W}{v},t\right)\varphi_0(v)\frac{dv}{v},$$

where $v=W/E$.

(b) Some photons initially in the interval (W, dW) are absorbed by pair production. According to Eq. (1.47a) their number is

$$dW\,dt \times \gamma(W,t)\sigma_0.$$

Therefore

$$\frac{\partial\pi(E,t)}{\partial t} = 2\int_0^1 \gamma\left(\frac{E}{u},t\right)\psi_0(u)\frac{du}{u} - \int_0^1\left[\pi(E,t) - \frac{1}{1-v}\pi\left(\frac{E}{1-v},t\right)\right]\varphi_0(v)dv, \tag{2.11}$$

$$\frac{\partial\gamma(W,t)}{\partial t} = \int_0^1 \pi\left(\frac{W}{v},t\right)\varphi_0(v)\frac{dv}{v} - \sigma_0\gamma(W,t). \tag{2.12}$$

The functions φ_0 and ψ_0 do not depend on the atomic number, hence the solutions of the Eqs. (2.11) and (2.12) are the same for all substances, provided, of course, we measure the thickness in radiation lengths. The functions φ_0 and ψ_0 depend only on the ratio between the primary energy and that of the emitted particle. Hence any solution of Eqs. (2.11), (2.12) remains valid if all energies are multiplied by a constant factor.

Comparison of cascades in matter

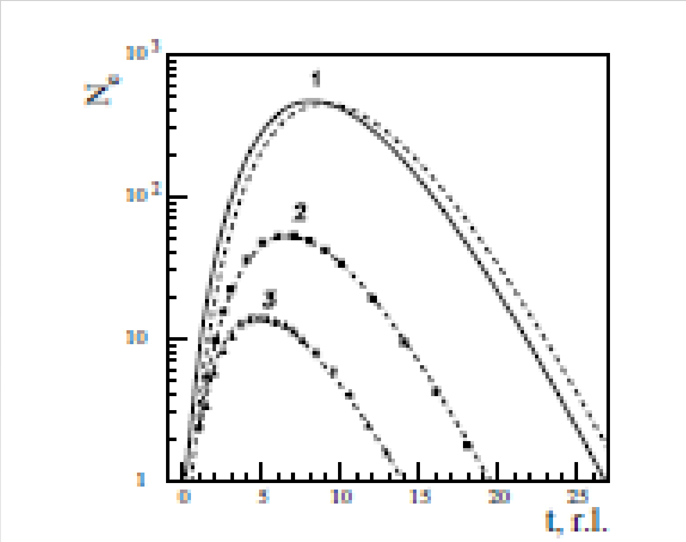


Fig. 7. Cascade curves of electrons for showers initiated by primary electrons (solid curves) and photons (dashed curves). The calculations are performed for the following primary energies $\varepsilon_0 = 2 \times 10^8$ (curve 1), 2×10^7 (curve 2), 2×10^6 (curve 3) and the ratio $\varepsilon_{th}/\varepsilon_{cr} = 1.25$ (curves 1 and 2), 0.05 (curve 3). For comparison, the results derived from the analytical cascade theory [1] (boxes) and by simulations with the ALTAI code [38] (triangles) are also shown.

Figure 3: Aharonian-Plyasheshnikov's Fig. 7.

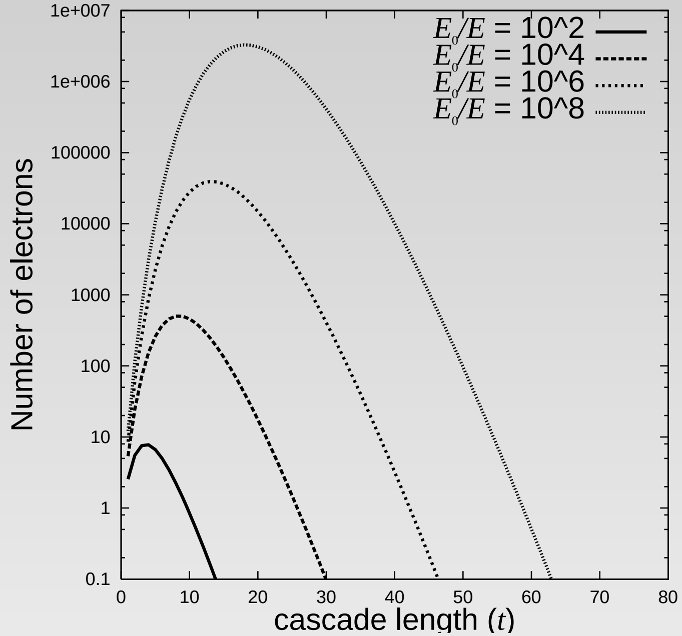


Figure 4: Our result

シャワーのピーク位置と値について、Nishimura の教科書にあるものを + マークで示し比較すると、

Numerically Integrated
shower in Matter under Appr. A

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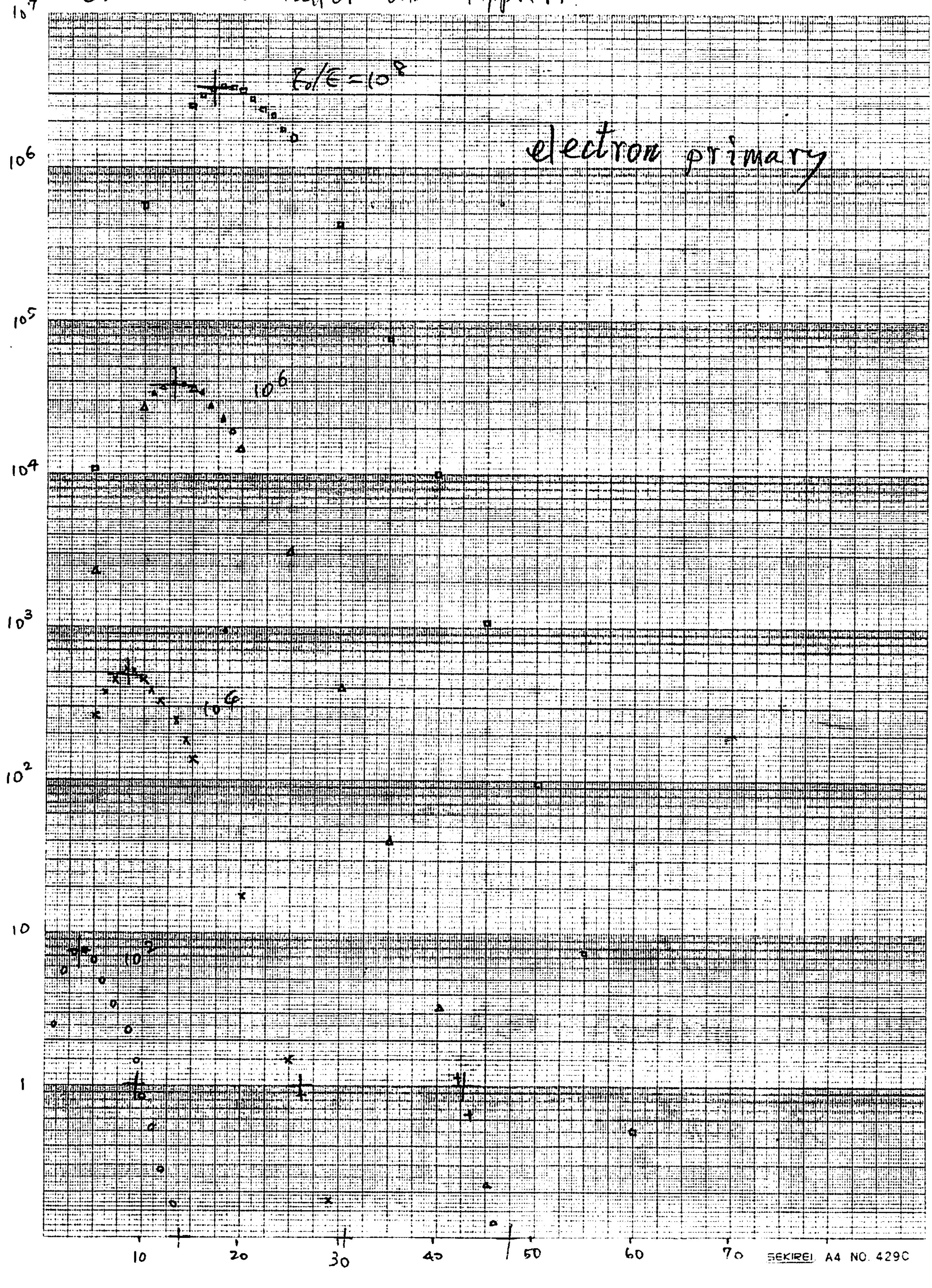


Our cascade results in photon fields

Diffusion equation

$$\begin{aligned} \frac{d}{\kappa_0 dt}\pi(\kappa,t) &= -\frac{\pi(\kappa)}{\kappa}\int_0^1 \phi(\kappa,v)dv + \int_\kappa^1 \phi(\kappa',1-\frac{\kappa}{\kappa'})\frac{\pi(\kappa')}{\kappa'}\frac{d\kappa'}{\kappa'} \\ &\quad + 2\int_\kappa^1 \psi(\lambda',\frac{\kappa}{\lambda'})\frac{\gamma(\lambda')}{\lambda'}\frac{d\lambda'}{\lambda'}, \\ \frac{d}{\kappa_0 dt}\gamma(\lambda,t) &= \int_\lambda^1 \phi(\kappa',\frac{\lambda}{\kappa'})\frac{\pi(\kappa')}{\kappa'}\frac{d\kappa'}{\kappa'} - \frac{\gamma(\lambda)}{\lambda}\int_0^1 \psi(\lambda,u)du, \end{aligned}$$

where

$$\kappa \equiv \omega_0\varepsilon_{\mathrm{e}} \qquad \text{and} \qquad \lambda \equiv \omega_0\varepsilon_\gamma,$$

and the energies are described in unit of mc^2.

The cross-sections are

$$\begin{aligned}\phi(\kappa, v) &= \frac{1}{4}\left(1 - v + \frac{1}{1-v}\right)\\ &+ \frac{v}{16\kappa(1-v)}\left(3 + v - \frac{1}{1-v} + 4\ln\frac{v}{4\kappa(1-v)}\right)\\ &- \frac{v^2}{16\kappa^2(1-v)^2},\\ \psi(s, u) &= \frac{1}{4}\left(\frac{1-u}{u} + \frac{u}{1-u}\right)\\ &- \frac{1}{16su(1-u)}\left(4 + \frac{1-u}{u} + \frac{y}{1-u} - 4\ln\left\{4su(1-u)\right\}\right)\\ &+ \frac{1}{16s^2u^2(1-u)^2},\end{aligned}$$

where the radiation length defined by Aharonian is ($\times$ 2 ?)

$$X_0^{(G)} = \left[4\pi n_0^{(G)} r_0^2\right]^{(-1)} \kappa_0.$$

The cross-sections for Inverse Compton (left) and photon-photon pair production (right) are indicated below:

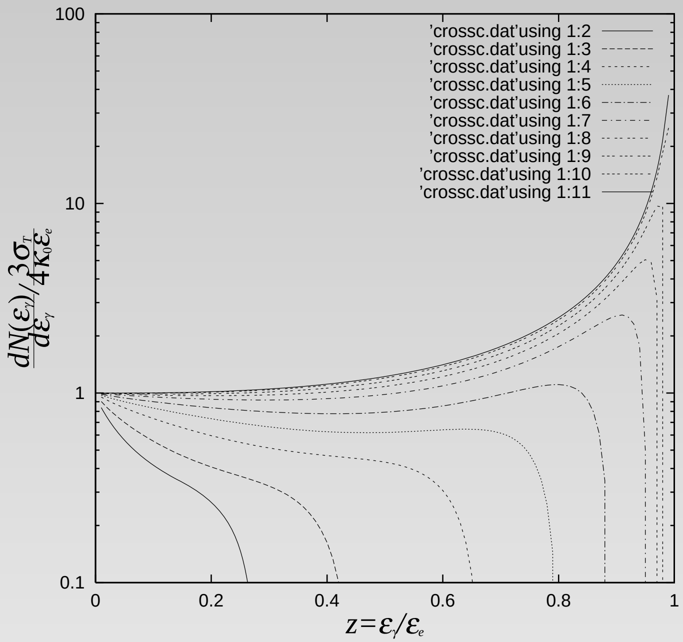


Figure 5: $\kappa_0 = .1, .2, .5, \cdots, 100,$ from left to right.

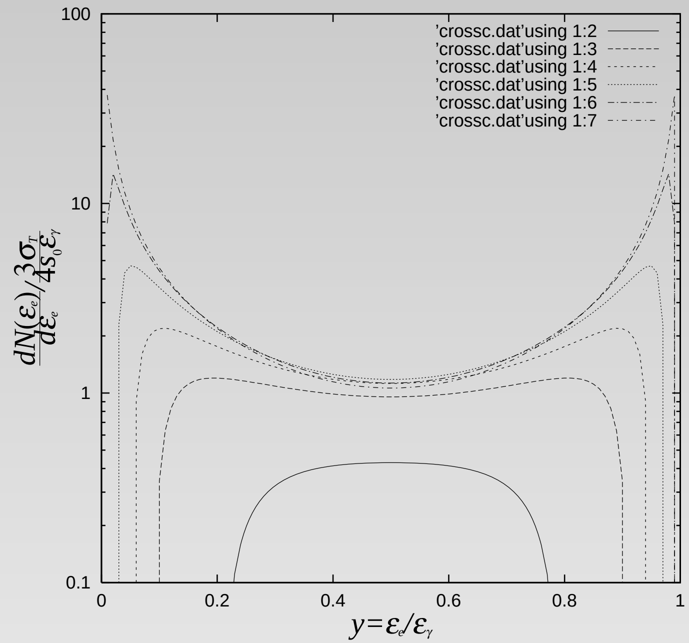


Figure 6: $\kappa_0 = .1, .2, .5, \cdots, 100,$ from bottom to top.

Comparison of cascades in photon fields

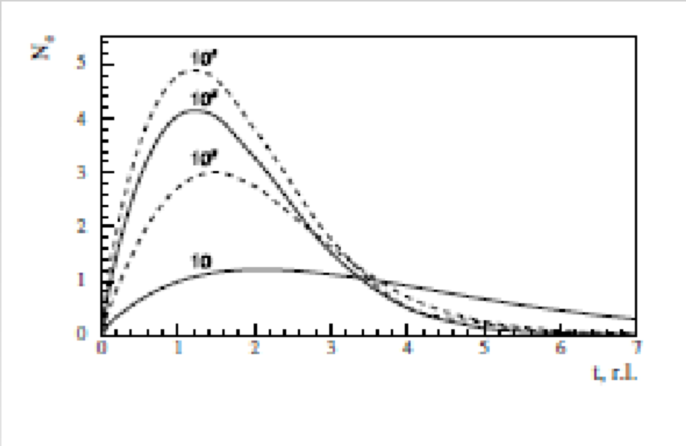


Figure 7: Aharonian-Plyasheshnikov's Fig. 9.

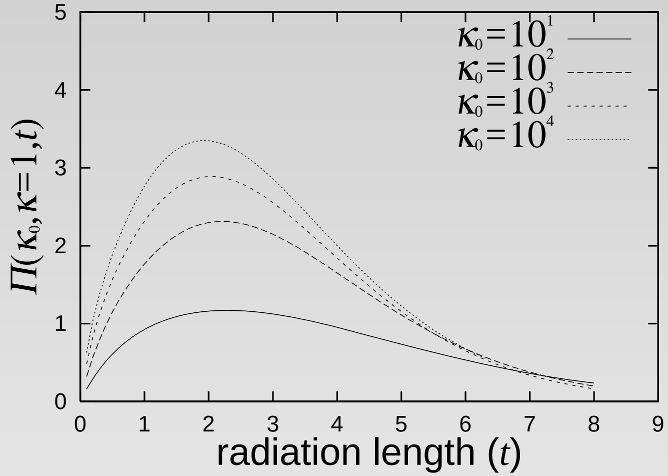


Figure 8: Our result

Aharonian-Plyasheshnikov の遷移曲線の結果をわれわれの結果と比較すると、

- 遷移曲線の形
- 入射エネルギーの増加に対する遷移曲線の変化の仕方
- ピーク位置が入射エネルギーによらない

ことなど、定性的にはよく一致するが、

- ピーク位置
- 最大粒子数

など、定量的な面では一致していない。

We discuss the differential energy spectrum

Aharonian-Plyasheshnikov result is

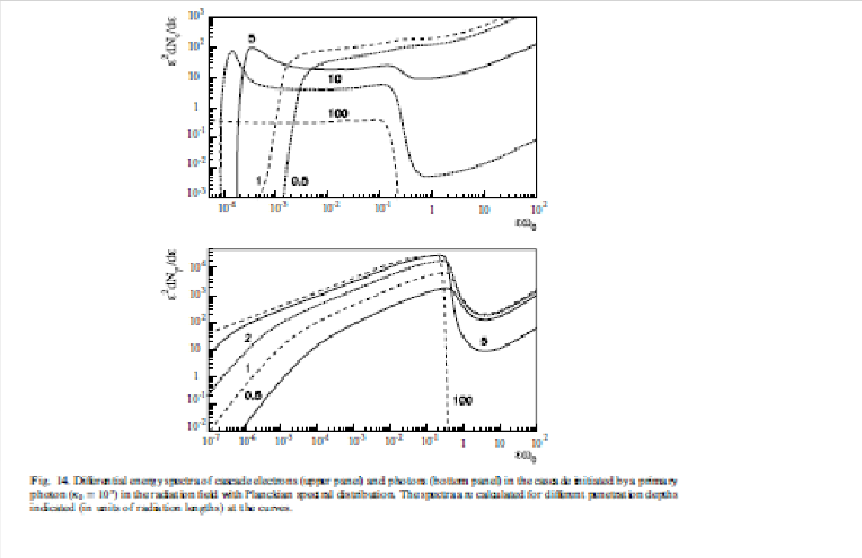


Figure 9: Aharonian-Plyasheshnikov’s Fig. 14.

and our results are

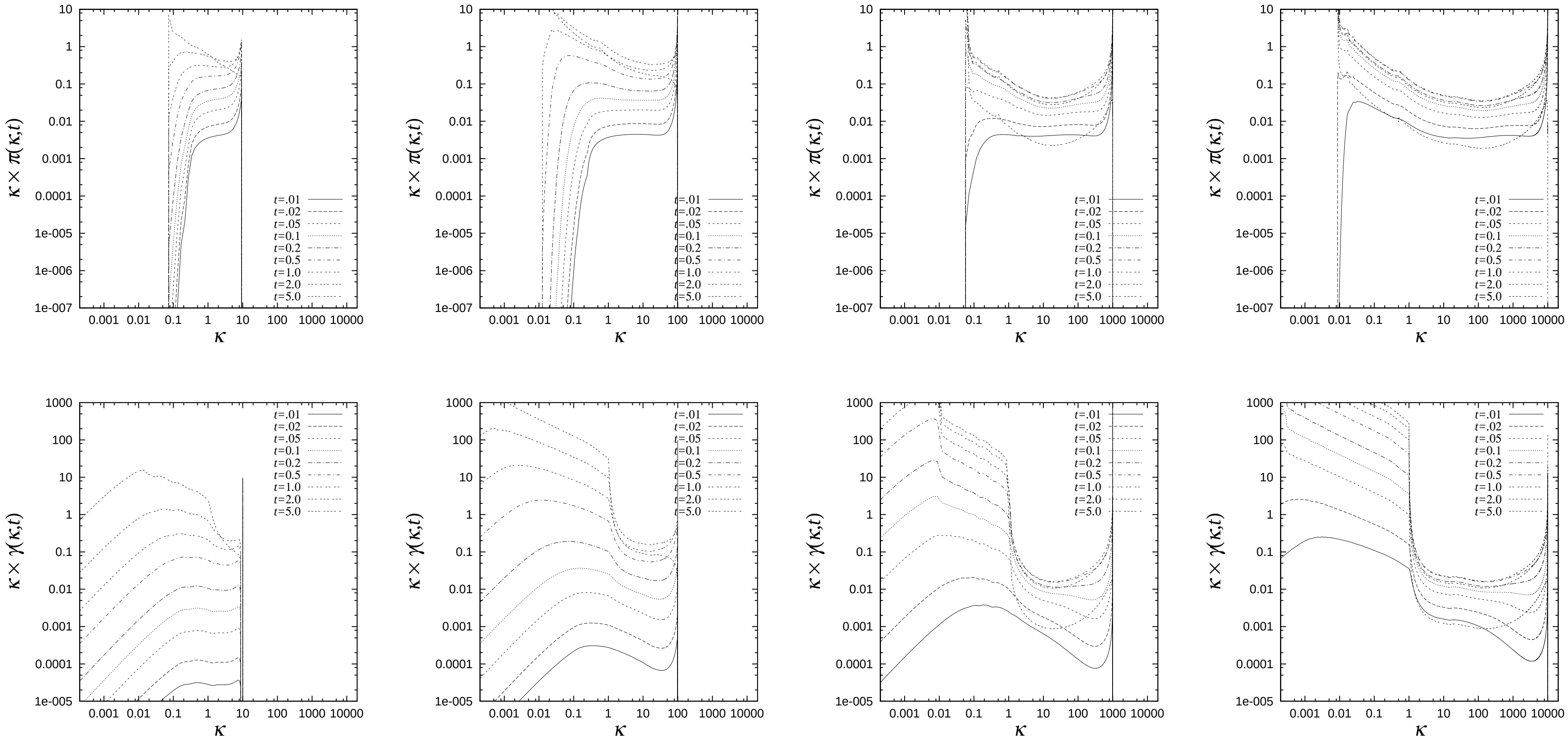


図 1: Diff. E spm of e (upper) and γ (lower). $\kappa_0 = 10$, $t = .01, .02, .05, .1, .2, .5, 1, 2, 5$.

Transition of cascades energy spectrum

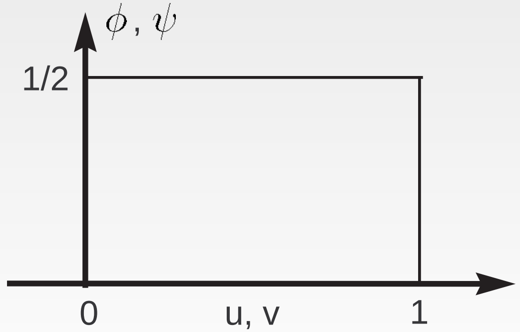
If we assume simple cross-sections

$$\phi(\kappa, v) = \frac{1}{2}, \qquad \text{and} \qquad \psi(\lambda, u) = \frac{1}{2},$$

the diffusion equations can be described as

$$\frac{\partial}{\partial t}\pi(\kappa,t) + \frac{\kappa_0}{2\kappa}\pi(\kappa,t) - \frac{\kappa_0}{2}\int_\kappa^{\kappa_0} \frac{\pi(\kappa',t)}{\kappa'^2}d\kappa' = \kappa_0 \int_\kappa^{\kappa_0} \frac{\gamma(\kappa',t)}{\kappa'^2}d\kappa',$$

$$\frac{\partial}{\partial t}\gamma(\kappa,t) + \frac{\kappa_0}{2\kappa}\gamma(\kappa,t) = \frac{\kappa_0}{2}\int_\kappa^{\kappa_0} \frac{\pi(\kappa',t)}{\kappa'^2}d\kappa'.$$



We search for the solutions corresponding to mono-energetic incident particle (Green's function). Applying Mellin transforms

$$\mathcal{M}(s,t) = \int_0^\infty \left(\frac{\kappa}{\kappa_0}\right)^s \pi(\kappa,t) d\kappa,$$

$$\mathcal{N}(s,t) = \int_0^\infty \left(\frac{\kappa}{\kappa_0}\right)^s \gamma(\kappa,t), d\kappa$$

then we get differential-difference equations

$$\frac{\partial}{\partial t}\mathcal{M}(s,t) + \frac{s}{2(s+1)}\mathcal{M}(s-1,t) = \left(\frac{\kappa'}{\kappa_0}\right)^s,$$

$$\frac{\partial}{\partial t}\mathcal{N}(s,t) + \frac{1}{2}\mathcal{N}(s-1,t) = \left(\frac{\kappa'}{\kappa_0}\right)^s.$$

The solutions for respective Mellin transforms are

$$\begin{aligned}\mathcal{M}(s,t) &= (\kappa'/\kappa_0)^s \left(1+\frac{\kappa_0 t}{2\kappa'(s+1)}\right) e^{-\kappa_0 t/(2\kappa')},\\ \mathcal{N}(s,t) &= (\kappa'/\kappa_0)^s e^{-\kappa_0 t/(2\kappa')},\end{aligned}$$

and applying inverse Mellin transforms, we have the respective Green's functions,

$$\begin{aligned}G_\pi(\kappa,t;\kappa') &= \left\{\delta(\kappa-\kappa')+\frac{\kappa_0 t}{2\kappa'^2}\right\} e^{-\kappa_0 t/(2\kappa')},\\ G_\gamma(\kappa,t;\kappa') &= \delta(\kappa-\kappa')e^{-\kappa_0 t/(2\kappa')}.\end{aligned}$$

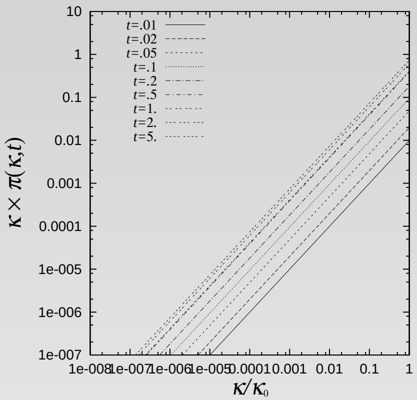
Using the Green's functions, we get the differential spectrum of the first and the second generation of electrons and photons,

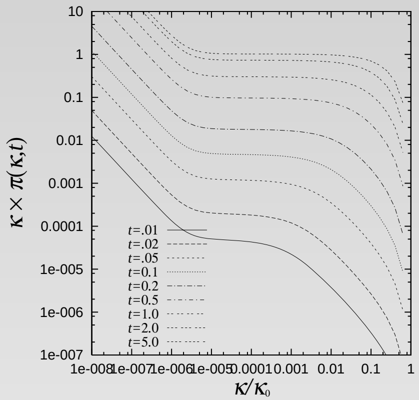
- $\gamma_1(\kappa, t)$, the incident photon spectrum (δ)
- $\pi_1(\kappa, t)$, the electrons spectrum produced by $\gamma_1(\kappa, t)$
- $\gamma_2(\kappa, t)$, the photons spectrum produced by $\pi_1(\kappa, t)$
- $\pi_2(\kappa, t)$, the electrons spectrum produced by $\gamma_2(\kappa, t)$

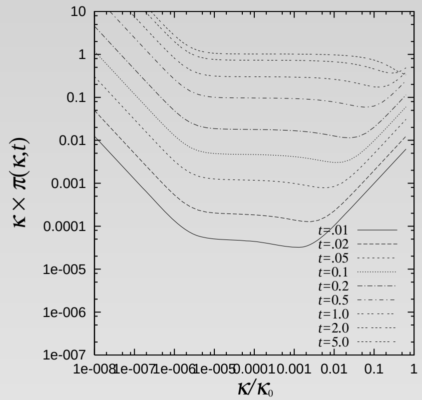
The solutions are

$$
\begin{aligned}
\gamma_1(\kappa,t) &= G_\gamma(\kappa,t;\kappa_0) = \delta(\kappa-\kappa_0)e^{-t/2}. \\
\pi_1(\kappa,t) &= \int_0^t dt' \frac{e^{-t'/2}}{\kappa_0} \int_\kappa^{\kappa_0} G_\pi(\kappa,t-t';\kappa')d\kappa' = \frac{t}{\kappa_0}e^{-t/2}, \\
\gamma_2(\kappa,t) &= \int_0^t dt' \frac{t'}{2}e^{-t'/2} \int_\kappa^{\kappa_0} \left(\frac{1}{\kappa'} - \frac{1}{\kappa_0}\right) G_\gamma(\kappa,t-t';\kappa')d\kappa' \\
&\simeq \frac{1}{4}\left(\frac{1}{\kappa} - \frac{1}{\kappa_0}\right) t^2 e^{-t/2} - \frac{\kappa_0}{24}\left(\frac{1}{\kappa} - \frac{1}{\kappa_0}\right)^2 t^3 e^{-t/2}, \\
\pi_2(\kappa,t) &= \kappa_0 \int_0^t dt' \frac{(t-t')^2}{8} e^{-(t-t')/2} \\
&\qquad \times \int_\kappa^{\kappa_0} \left(\frac{1}{\kappa'} - \frac{1}{\kappa_0}\right)^2 G_\pi(\kappa,t';\kappa')d\kappa' \\
&= \frac{1}{2\kappa_0} \int_0^t \frac{1}{t'^2} \left\{ 2e^{-t'/2} - \left(2 - t' + \frac{t'}{v}\right) e^{-t'/(2v)} \right\} \\
&\qquad \times (t-t')^2 e^{-(t-t')/2} dt',
\end{aligned}
$$

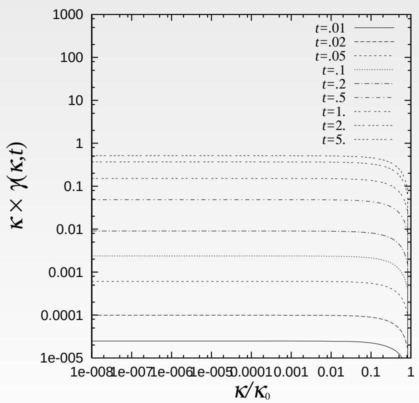
The differential electron spectra $\pi_1(\kappa, t)$, $\pi_2(\kappa, t)$, and $\pi_1(\kappa, t) + \pi_2(\kappa, t)$ are indicated in the left, the middle, and the right.







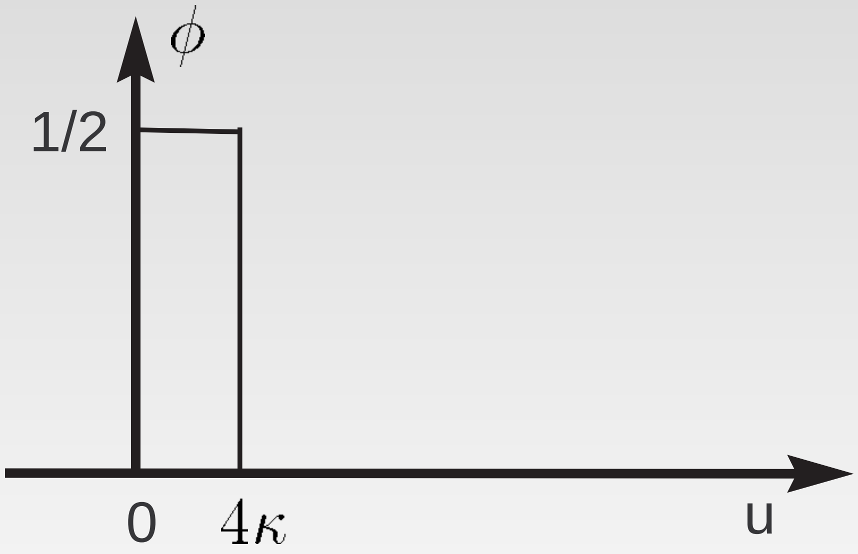
and, the differential photon spectrum $\gamma_2(\kappa, t)$ is indicated.



Electrons' cooldown process

At electron energies of $\kappa \ll 1$, maximum fraction of radiation energy by Inverse Compton becomes suppressed, as

$$\phi(\kappa, v) \simeq \frac{1}{2}, \qquad \text{where} \qquad v < 1/\left(1 + \frac{1}{4\kappa}\right) \simeq 4\kappa,$$



then electrons diffuse as

$$\begin{aligned}
\frac{\partial}{\kappa_0 \partial t}\pi(\kappa,t) &= \frac{1}{\kappa}\int_0^1 \left\{ \phi(\frac{\kappa}{1-v},v)\pi(\frac{\kappa}{1-v},t) - \phi(\kappa,v)\pi(\kappa,t) \right\} dv \\
&\simeq \int_0^1 v\frac{\partial}{\partial \kappa}\left\{\phi(\kappa,v)\pi(\kappa,t)\right\} dv \simeq 8\kappa\pi(\kappa,t) + 4\kappa^2\frac{\partial}{\partial\kappa}\pi(\kappa,t).
\end{aligned}$$

Applying Mellin transforms, we have

$$\frac{\partial}{\partial t}\mathcal{M}(s,t) + 4\kappa_0^2 s\mathcal{M}(s+1,t) = 0.$$

The solution for the initial condition $\pi(\kappa,0) = \delta(\kappa-\kappa')$ is

$$\mathcal{M}(s,t) = \left(\frac{\kappa_0}{\kappa'} + 4\kappa_0^2 t\right)^{-s},$$

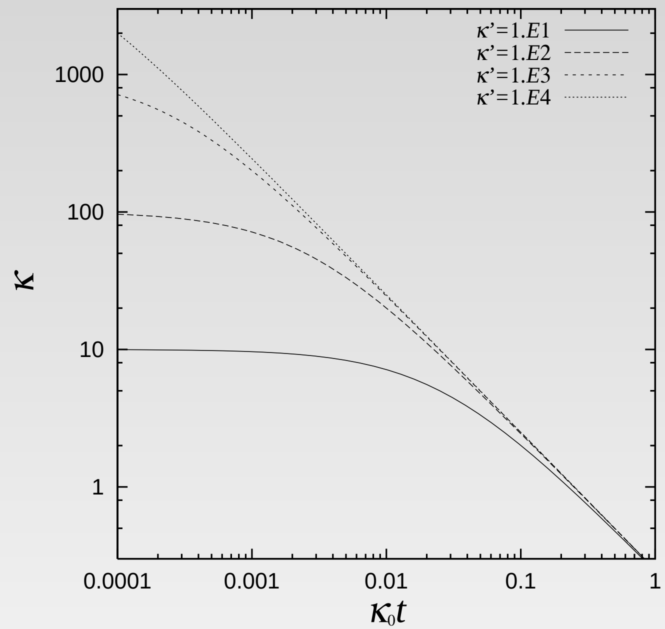
so, applying inverse Mellin transforms we have

$$\pi(\kappa,t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{{\kappa_0}^s}{\kappa^{s+1}}\left(\frac{\kappa'}{\kappa_0 + 4\kappa_0^2 t\kappa'}\right)^s ds = \delta(\kappa - (\frac{1}{\kappa'} + 4\kappa_0 t)^{-1}).$$

This solution indicates that electron of the initial energy κ' decrease its energy κ as

$$\frac{1}{\kappa} = \frac{1}{\kappa'} + 4\kappa_0 t,$$

after traversing the thickness of t, [a] as indicated in the figure.



[a] This relation corresponds to the solution from the mean energy dissipation, $-\frac{d}{\kappa_0 dt}\kappa = \int_0^1 v\phi(\kappa, v)dv \simeq \frac{1}{2}\int_0^{4\kappa} vdv = 4\kappa^2$.

And under the electron cooldown process, photons diffuse as

$$\frac{\partial}{\kappa_0 \partial t}\gamma(\lambda, t) = \int_{\lambda}^{\kappa_0} \phi(\kappa', \frac{\lambda}{\kappa'}) \frac{\pi(\kappa', t)}{\kappa'} \frac{d\kappa'}{\kappa'} \qquad (\lambda < 1).$$

Fractional inverse Compton radiation is suppressed as

$$\frac{\lambda}{\kappa'} < 1/\left(1 + \frac{1}{4\kappa'}\right), \quad \text{so that} \quad \kappa' > (\lambda + \sqrt{\lambda^2 + \lambda})/2.$$

$(\lambda + \sqrt{\lambda^2 + \lambda})/2 \simeq \sqrt{\lambda}/2$ shows the lower bound of the integral.

Electrons cool down very slowly, so that they have minimum limit $\kappa_{\min}$ in their energy spectrum.

When the lower bound of integral exhists in electron spectrum, or $\sqrt{\lambda}/2 > \kappa_{\rm min}$,

$$\frac{\partial}{\kappa_0 \partial t}\gamma(\lambda, t) = \int_{(\lambda+\sqrt{\lambda^2+\lambda})/2}^{\kappa_0} \frac{\pi(\kappa')}{2\kappa'}\frac{d\kappa'}{\kappa'},$$

so for the power-type electron spectrum $\pi(\kappa, t) \simeq \kappa^{-\alpha} f(t)$,

$$\frac{\partial}{\kappa_0 \partial t}\gamma(\lambda, t) \simeq \frac{f(t)}{2(\alpha+1)}\left(\frac{\lambda+\sqrt{\lambda^2+\lambda}}{2}\right)^{-\alpha-1} \simeq \frac{2^\alpha}{\alpha+1} f(t)\lambda^{-(\alpha+1)/2}.$$

As electrons show power index of $\alpha = 2$ in cooldown process, photon spectrum shows $\lambda^{-3/2}$.

On the other hand when the lower bound of the integral is smaller than the electron spectrum, or $\sqrt{\lambda}/2 < \kappa_{\rm min}$, photon spectrum $\gamma(\lambda, t)$ becomes independent on their energy λ, as already predicted by Aharonian and Plyasheshnikov.

Conclusions

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- Our results of cascades in matter and the strong magnetic fields agreed well with Aharonian and Plyasheshnikov results, and showed enough accuracies.
- We have not yet obtained quantitatively consistent results with Aharonian and Plyasheshnikov's for cascades in photon fields.

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- Our results of cascades in matter and the strong magnetic fields agreed well with Aharonian and Plyasheshnikov results, and showed enough accuracies.
- We have not yet obtained quantitatively consistent results with Aharonian and Plyasheshnikov's for cascades in photon fields.
- Transition of the diffferential energy spectrum is well explained by the differntial-difference equation with the simplified cross-sections and the electron cooldown process.